REVIEW

1. Cost Concepts

Review of Cost Concepts

C(y) = minimal expenditure required to produce output y.

Two components: fixed costs F, and variable costs V(y).

$$C(y) = F + V(y)$$

Fixed costs do not vary with output.

Variable costs increase with output.

Also two kinds of fixed costs: sunk and not sunk. Fixed costs are sunk if they cannot be avoided even if firm shuts down (i.e., output is zero); they are not sunk if firm has to pay them only if output is positive.

$$C(y) = \begin{cases} F_0, & y = 0\\ F_0 + F_1 + V(y), & y > 0 \end{cases}$$

Examples: cell phone towers, allocation of a plane to a city pair market.

C'(y) = MC(y): marginal cost C(y)/y = AC(y): average cost V(y)/y = AVC(y): average variable cost. Example: $C(y) = 1 + y^2, y \ge 0$

Fixed Cost: 1 (sunk since C(0) = 1) Total Variable Cost: y^2 Average Cost: $\frac{1}{y} + y$ Average Variable Cost: yMarginal Cost: 2y

Review of Supply Under Perfect Competition

Basic Assumption: firm's objective is to maximize profits.

$$\pi(y)=R(y)-C(y)$$

Here R(y) is revenue from selling y units; MR(y) = R'(y).

Let y^* denote the profit-maximizing output. If $y^* > 0$, then it solves $\pi'(y) = 0$ or, equivalently

$$MR(y) = MC(y)$$

We assume R is concave, C is convex. Under perfect competition, firms cannot affect the price P. That is,

R(y) = Py and MR(y) = P Thus, if y^* is positive, then it satisfies

$$P = MC(y^{\star})$$

It is in this sense that the competitive firm's (inverse) supply curve is its marginal cost curve.

When is $y^* > 0$? That is, when will the firm choose to operate?

Need to check that the firm's profits from operating exceed its profits from shutting down.

Let z denote the output level that minimizes AC(y). Recall that z solves MC(y) = AC(y); i.e.,

Total
$$Cost(y) = AC(y) \times y$$

where Marginal $Cost \equiv \frac{dTC(y)}{dy}$
 $\Rightarrow MC(y) = \frac{dAC(y)}{dy} \times y + AC(y)$ [Product Rule]

When does MC(y) = AC(y)?

$$\underbrace{\frac{dAC(y)}{dy} \times y + AC(y)}_{dy} = AC(y)$$
$$\Rightarrow \frac{dAC(y)}{dy} = 0$$

Formally, this is true when $y \neq 0$ (i.e., the only interesting case) and we should also check second-order condition to make sure this is minimum and not a maximum. Q: What kind of cost function would imply an inverse-U (i.e., strictly-concave) average cost curve?

Case 1: Not Sunk Fixed Costs

Case 1: If fixed costs are not sunk, then $\pi(0) = 0$. In this case, the firm makes positive profits from operating only if

$$\pi(y) = Py - C(y) > 0 \rightarrow P > AC(y)$$

In words, it operates at any price above the minimum of average cost and shuts down when price is below this minimum. Thus,

$$y^{\star} = 0$$
 if $P < AC(z)$

If P > AC(z), then y^* solves P = MC(y). We refer to AC(z) as the shutdown point. In words, the supply curve is the part of the marginal cost curve that lies above shutdown point.

Case 2: Sunk Fixed Costs

Case 2: If fixed costs are sunk, then $\pi(0) = -F$. In this case, the firm cannot avoid the fixed costs by shutting down so it operates at any positive price. The shutdown price is 0 and the firm's supply curve is its marginal cost curve.

Learning Check-Point

 $\frac{\text{Example 1 (cont.):}}{C(y) = 1 + y^2, y \ge 0.}$

- Derive the firm's supply curve.
- Are fixed costs sunk or not sunk?

Entry and Exit

In **short-run**, number of firms and their investments in capital are fixed. The investments in capital (e.g., plants, cell towers) are fixed costs that are typically sunk and cannot be avoided.

- If P < min AC(y), firms operate because they can cover variable costs. But they are losing money since they cannot cover fixed costs.
- If $P > \min AC(y)$, firms operate and are making positive economic profits.

In the **long-run**, capital is not sunk since it depreciates and needs to be replaced. All costs (i.e., inputs) are avoidable.

- If $P < \min AC(y)$, firms will not replace capital and exit.
- If $P > \min AC(y)$, outside firms will enter the industry.

Equilibrium

A **Competitive Equilibrium** is a triplet of price, output, and number of firms (i.e., p, y, N) characterized by three conditions:

- 1. (π max.) Firms maximize profits $\rightarrow P^{\star\star} = MC(y^{\star\star})$
- 2. (free entry) Firms make zero economic profits $\rightarrow P^{\star\star} = AC(y^{\star\star})$
- 3. (markets clear) Supply is equal to demand $\rightarrow D(P^{\star\star}) = S(P^{\star\star})$

Note: Industry supply is sum of the individual firm supply.

N.b., In this class, the concept of an "equilibrium" will be very important. Often we will find answers to our research / motivating questions by analyzing how the equilibrium changes as we change something in the model.

Learning Check-Point

Example 2:

D(P) = 136 - P; $C(y) = 32 + 0.5 \times y^2$ if y > 0.

Solve for the Competitive Equilibrium.

2. Welfare

We define the social welfare (W) at price P as the sum of market consumer surplus (CS) and market producer surplus (PS).

$$W(Y(P)) = CS(Y(P)) + PS(Y(P))$$

Deep Thought: Welfare is defined by production and consumption of goods (or services) where P is the means by which the producer and consumer exchange these goods (or services). Money itself, and therefore its price P, is inherently worthless except that it facilitates exchange of goods (or services).

Consumer Surplus

Consumer surplus: difference between what the consumer is willing to pay and what she has to pay.

If she purchases y units at price P per unit, then her consumer surplus is

$$CS_k(y, P) = T_k(y) - Py$$

where $T_k(y)$ is the area under the demand curve when consumer k buys y units. Note that this is equivalent to the amount he/she would be willing to pay for y units (Why?).

Utility-maximization implies she should purchase y units up to the point where her marginal willingness to pay is equal to the price, that is

$$T_k'(y) = P$$

Thus, given P we have an implicit function to tell us quantity demanded y by consumer k.

Consumer Surplus, cont'd

Let $y_k(P)$ be the solution to the above equation: it is consumer k's demand at price P. Then her consumer surplus at price P is

$$CS_k(P) = \underbrace{T_k\left(y_k(P)\right)}_{T_k(P)} - Py_k(P)$$

Here $T_k(P)$ is the area under consumer k's demand curve and $CS_k(P)$ is the area under her demand curve and above the price.

The market consumer surplus at price P is simply the sum of the individual consumer surpluses: $CS(P) = \sum_{k=1}^{K} CS_k(P)$.

Producer Surplus

Producer surplus: difference between what the producer gets from selling and her costs of production.

If producer i sells y units at price P, then its producer surplus is

$$PS_i(y, P) = Py - C_i(y)$$

Under PC each producer produces only up to the point where marginal cost is equal to price, that is,

$$P = C_i'(y)$$

Again, given P we have an implicit function to tell us quantity supplied y.

N.B., this is true even when firms have market power and choose output s.t. MR(y) = MC(y)

Producer Surplus, cont'd

Let $y_i(P)$ denote the solution to the above equation: it is her supply function. Then producer *i*'s producer surplus (or profits) at price P is

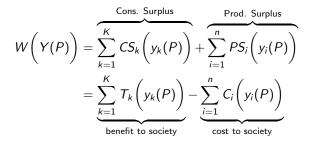
$$PS_i(P) = Py_i(P) - C_i\left(y_i(P)\right)$$

**PS_i*(*P*) is not necessarily equal to $\pi_i(P)$ since *PS*(*P*) ignores sunk fixed costs.

The market producer surplus at price *P* is simply the sum of the producers' surpluses: $PS(p) = \sum_{i} PS_{i}(p)$.

Welfare

Welfare is then



We say that "welfare at price P is the difference between the amount that consumers are willing to pay at price P and the costs of producing those units."

An Aside: What's the Optimal Y?

- ► Forget about price P for a moment and for simplicity set K = 1 (one person) and I = 1 (one firm).
- Ask yourself: How much Y should be produced if we want to maximize welfare?

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- ► Forget about price P for a moment and for simplicity set K = 1 (one person) and I = 1 (one firm).
- Ask yourself: How much Y should be produced if we want to maximize welfare?
- If we assume that W(Y) is strictly-concave (i.e., a hill), optimal Y solves:

$$\max_{Y} \left\{ \underbrace{T(Y) - C(Y)}_{W(Y)} \right\}$$

The first-order condition (FOC) is:

$$\frac{dT(Y)}{dY} = \frac{dC(Y)}{dY}$$

So Welfare is maximized when the marginal utility to consumers (LHS) is equal to the marginal cost of production (RHS).

Back to the Competitive Equilibrium

Important points:

1. The payments that consumers make to producers are a transfer and do not count towards welfare.

2. In competitive markets, the consumer's marginal willingness to pay at P is equal to the producer's marginal cost:

Consumers:
$$\frac{dT(y)}{dy} = P$$

Firms: $P = \frac{dC(y)}{dy}$ *i.e.*, "price = mc"
 $\Rightarrow \frac{dT(y)}{dy} = \frac{dC(y)}{dy}$

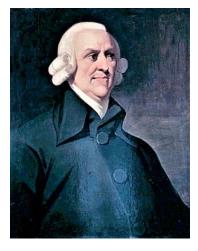
*This means that at price P the benefit to society is just offset by the cost to society.

Hence, the allocation maximizes social welfare!

Comments

In words, the amount each consumer is willing to pay for the last unit she purchases is equal to the cost of producing that unit. Hence, there are no gains from trading more.

Free markets maximize welfare! \rightarrow Adam Smith's "Invisible Hand."



- Adam Smith (1723-1790)
- Scottish economist and philosopher.
- Key figure during the Scottish Enlightenment.
- Best known for

An Inquiry into the Nature and Causes of the Wealth of Nations (1776)

which is considered to be the first modern work of economics.

What factors have we not modeled which might overturn this result?

- 1. Externalities (i.e., "missing markets").
- 2. Market power.

Learning Check-Point

Example 3:

Ten consumers, unit demands; $T_k(y) = 11 - k$. Assume that marginal costs are zero.

Solve for the equilibrium price, consumer surplus, and producer surplus and show that it maximizes total welfare (aka "total surplus").

Hint: Solve for the equilibrium price and then identify which of the ten heterogenous consumers choose to buy (i.e., have $CS_k(p^*) \ge 0$). Then solve for $CS(p^*) = \sum_{k=1}^{10} CS_k(p^*)$.

Example 4:

Linear demand (you choose the specification), zero marginal costs.

N.b., We'll often assume that costs are zero in order to focus our attention on the effects of a particular firm behavior on total revenue. Allowing for more interesting cost functions then amounts to an extension.

3. Demand

Review of Demand

We define D_k(P) as quantity demanded by consumer k at price P. For example, consider the case when a, b > 0 and linear demand:

$$Y_k = a_k - b_k P$$
 ("Demand")
 $P = rac{a_k}{b_k} - rac{Y_k}{b_k}$ ("Inverse Demand")

The "Price Elasticity of Demand" is defined as

$$\epsilon(P) \equiv \frac{\%\Delta Y(P)}{\%\Delta P} = \frac{\frac{\Delta Y(P)}{Y(P)}}{\frac{\Delta P}{P}} = \frac{\Delta Y(P)}{\Delta P} \times \frac{P}{Y(P)}$$

- N.B., when changes are small, $\frac{\Delta Y(P)}{\Delta P} = \frac{dY}{dP}$.

Demand Elasticity with Linear Demand Example: Suppose Y(P) = a - bP where b > 0 then $\frac{dY}{dP} = -b$ and $\epsilon(P) = -b \times \frac{P}{Y(P)}.$ P Elastic **Unit Elastic** Inelastic C

4. Monopoly

Regulation of Monopoly – Sherman Antitrust Act of 1890

Sherman Act, Section 1:

"Every contract, combination in the form of a trust or otherwise, or conspiracy, in restraint of trade or commerce among several states, or with foreign nations, is declared to be illegal."

Sherman Act, Section 2:

"Every person who shall monopolize, or attempt to monopolize, or combine or conspire with any other person or persons, to monopolize any part of the trade or commerce among the several States or with foreign nations, shall be deemed guilty of a felony".

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- Prohibits anti-competitive mergers or takeovers
- "per se" illegal action is inherently illegal.
- "rule of reason" action may be okay if the effect is not bad.

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- Prohibits anti-competitive mergers or takeovers
- "per se" illegal action is inherently illegal.
- "rule of reason" action may be okay if the effect is not bad.

A monopoly is not illegal per se.

Research Questions

- 1. Why do we have laws against monopolies?
- 2. Are they really "bad" for society?

Approach

- a. Construct a theory of firm behavior when it can control price.
- b. Solve for the monopoly equilibrium.
- c. Use the model and equilibrium to identify general patterns in firm behavior under market power.
- d. Compare welfare generated under the monopoly equilibrium to welfare-maximizing equilibrium (i.e., the Competitive Equilibrium) to see if society is "worse off".

Definition: A firm is a monopoly if it is the **only** supplier of a product in a market.

 \Rightarrow the firm is not a price-taker, rather its choices determine market price.

Why monopoly?

- Patents
- Natural monopoly

Natural Monopolies

Definition: An industry is a Natural Monopoly (NM) with respect to quantity Y if the least expensive way to produce Y is by one firm. That is, for any quantities,

$$y_1 + \ldots + y_n = Y$$

$$C(Y) \leq C(y_1) + \ldots + C(y_n)$$

It is NM in some range of quantities if it is a NM for every quantity in that range (U-shaped).

Examples: C(Y) = F + cY; $C(Y) = F + cY^2$.

* If AC is decreasing for all y, then industry is a NM. If AC is increasing for all y, then it may be a NM.

Applications: telephone, cell phone, electricity.

How Will the Monopolist Behave?

- Suppose a monopolist maximizes profit by choosing output (y).
- It faces a trade-off when it increases output:
 - 1. More output can only be sold by lowering the price.
 - 2. The reduction in price reduces revenue on infra-marginal (existing consumers) units but increases revenue on the extra marginal unit (new consumers).

A Calculus-free Example

Example 1: Marginal costs are zero. Demand is

$$y(p) = \begin{cases} 0, & \text{if } p > 4 \\ 10, & \text{if } 1$$

Two Interpretations:

- Each consumer wants only one unit; ten consumers are willing to pay \$4, ten consumers are willing to pay only \$1.
- Ten identical consumers, each have a willingness to pay of \$4 for first unit, \$1 for second unit.

Comparing the Monopoly and Competitive Equilibria

Monopoly solution: $p^{M} = 4, y^{M} = 10, \pi^{M} = 40, CS = 0, W = 40.$

Basic Point: Price is above marginal cost so allocation is not efficient and the gains from trade are not exhausted.

- Competitive welfare $= 4 \cdot 10 + 1 \cdot 10 + 0 = 50
- Dead Weight Loss = \$50 \$40 = \$10

General Case of Continuous (and Differentiable) Demand

Monopolist's profit maximization problem:

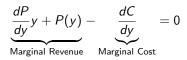
$$\max_{y} \pi(y) \equiv \max_{y} \left\{ P(y)y - C(y) \right\}$$

which is equivalent to

$$\max_{p} \pi(p) = \max_{p} \left\{ Py(p) - C(y(p)) \right\}$$

Remark: choosing y or P makes no difference since monopolist is selecting a single point on a strictly-decreasing demand curve.

F.O.C. is:



This is the familiar MR = MC rule.

The "Inverse-elasticity Rule"

It can also be expressed as follows:

$$\frac{\mathsf{P}-\mathsf{MC}}{\mathsf{P}}=\frac{-1}{\epsilon(\mathsf{P})}>0$$

This is known as the *inverse-elasticity rule* for the monopolist.

Key Result: price exceeds marginal cost; someone's willingness to pay for another unit exceeds the cost of producing that unit \rightarrow gains from trade are possible.

Remark: Monopolist always operates on elastic portion of demand curve.

Learning Check-Point

Example 2: Costs are zero. Ten identical consumers, each with continuous demand

$$y_k(P) = 1 - P, \ k = 1, ..., 10$$

Compare the competitive and monopoly equilibria.

Learning Check-Point

Example 2: Costs are zero. Ten identical consumers, each with continuous demand

$$y_k(P) = 1 - P, \ k = 1, ..., 10$$

Compare the competitive and monopoly equilibria. Competitive Equilibrium:

•
$$P = 0, Y^{C} = 10, CS^{C} = 5, PS^{C} = 0, W^{C} = 5.$$

Monopoly Solution

Need to find profit-maximizing price (or output) for the firm. Firm solves

$$\max_{Y} \left\{ \underbrace{\left(1 - \frac{Y}{10}\right)}_{P(Y)} \times Y - \underbrace{0}_{C(Y)} \right\}$$

where

$$\underbrace{Y(P) = 10 \times (1 - P)}_{\text{"Demand"}} \Rightarrow \underbrace{P(Y) = \frac{10 - Y}{10}}_{\text{"Inverse-demand"}}$$

Monopoly Solution (cont'd)

$$\frac{d\pi}{dY} \equiv 1 - \frac{Y}{10} + \left(\frac{-1}{10}\right) Y = 0$$
$$Y^{M} = 5, P^{M} = \frac{1}{2},$$
$$CS^{M} = 10 \times \frac{(.5 \cdot .5)}{2} = 5/4,$$
$$PS^{M} = 10 \times (.5 \cdot .5) = 5/2.$$

Monopoly Equilibrium:

$$\blacktriangleright W^{C} = 5, W^{M} = 15/4 \Rightarrow DWL = 5/4.$$

How Does a Monopoly Respond to a Change in Costs?

- Suppose the marginal cost of a firm changes by $\Delta c > 0$ (*i.e.*, costs increase).
- If the firm operates in a perfectly competitive industry, market clearing price increases by Δc and all of the change in cost is borne by consumers.
- ▶ In such a scenario, we say "pass-through" is complete.
- When the firm has market power, however, it is unclear what the pass-through rate is.

How Does a Monopoly Respond to a Change in Costs?

Recall the firm's FOC which summarizes its optimal choice of price:

$$\underbrace{Y(P) + (P-c) \times \frac{dY(P)}{dp}}_{\frac{d\pi}{dp}} = 0.$$
 (1)

Totally-differentiating (1) wrt p and c yields the following (work on next slide):

$$egin{aligned} rac{dp}{dc} &= rac{Y_p}{2Y_p - Y imes Y_{pp}/Y_p} \ &= rac{1}{2-\sigma} > 0 \,. \end{aligned}$$

where Y_p and Y_{pp} are the first and second derivatives of the demand function with respect to price and $\sigma(P) \equiv Y(P) \times Y_{pp}(P)/[Y_p(P)]^2$. We call $\sigma(P)$ the "curvature" of demand (at price P).

- ▶ Notice that pass-through is always positive, incomplete for $\sigma < 1$, and more than complete when $\sigma > 1$. As an aside, $\sigma < 2$ ensures the profit function is strictly concave in price.
- N.b., Demand "curvature" and "elasticity" are not generally the same thing.

Total Differentiation of the Firm's FOC

We totally-differentiated the firm's FOC

$$Y(P) + (P - c) \times Y_{\rho}(P) = 0$$
(N.b., $Y_{\rho}(P) \equiv \frac{dY(P)}{d\rho}$) to get
$$dp \left[Y_{\rho}(P) + Y_{\rho}(P) + (P - c) \times Y_{\rho\rho}(P) \right] + dc \left[-Y_{\rho}(P) \right] = 0$$

Rearrange and simplify to get

$$\frac{dp}{dc} = \frac{Y_p(P)}{2Y_p(P) + (P-c) \times Y_{pp}(P)}$$

▶ The "inverse elasticity rule" implies $P - c = Y(P)/Y_p(P)$ (you should check this!) so we can simplify further

$$rac{dp}{dc} = rac{Y_p}{2Y_p + Y imes Y_{pp}/Y_p}$$

and the result follows by plugging in the definition of σ .

Also not that this is just an example of the "Implicit Function Theorem" where the function g was the firm's FOC:

$$g(c,p) = Y(P) + (p-c) \times Y_p(P) .$$

Conclusions

- 1. Monopolies restrict output and increase prices relative to perfect competition.
- 2. Cost pass-through depends upon the curvature of demand, not upon the demand elasticity.
 - Firms pass-on some (and maybe) more of an increase in costs to consumers.
 - Consumer demand (via curvature) disciplines the change in prices.
- 3. Total surplus is maximized at the perfectly competitive solution; the monopoly solution leaves some gains from trade unrealized.
- 4. Policy Implication: try to eliminate or reduce the deadweight loss.

Remarks:

Monopoly is "bad" anytime society gives weight consumers.
 ⇒ regulation may be welfare improving.

Partial equilibrium - maximizing total surplus in one market may not be desirable if surplus is not maximized in other markets.

 \Rightarrow If a government uses a subsidy to correct behavior, it raises the subsidy through taxation which may generate a bigger distortion.

Two options:

- 1. Break up the monopoly by encouraging competition (*i.e.*, antitrust)
- 2. Dictate firm actions directly (e.g., price controls)

An Application of Anti-trust Law

Motivation - US vs Aluminum Company of America (1945)

The Charge:

ALCOA monopolized the market for aluminum ingot, violating Section 2 of the Sherman Act.

Background:

- Initially (1889) a monopoly due to patents on manufacturing processes; patent expired in 1906.
- Engaged in unlawful exclusionary and restrictive contracts to maintain monopoly control during period 1906-1912.
- From 1912-1940, it maintained its "monopoly" status by preemptive capacity investments to meet growing demand.

Questions

1. Is ALCOA a monopoly?

We need to define the market and then compute market share. Three possible definitions:

- a. Virgin ingot sales + secondary ingot: 33%
- b. Virgin ingot production + secondary ingot: 64%
- c. Virgin ingot production: 90%

Chief Judge Learned Hand (yes, "Learned Hand" was his actual name) creates the following rule-of-thumb for determining whether a firm is a monopoly: 90% is monopoly, 60% is doubtful, and 30% is definitely not monopoly.

Judge Hand argues for (c).

- Includes ALCOA's production of virgin ingot for its own fabricating plants because own demand competes with market demand.
- Excludes secondary ingot because ALCOA controls current supply of secondary through past production of virgin ingot.

Hand also discards the lack of profitability defense.

"The mere fact that a producer, having command of the domestic market has not been able to make more than a fair profit is no evidence that a fair profit could not have been made a lower prices."

2. Is ALCOA an illegal monopoly?

Remember, a monopoly is not illegal if it cannot be avoided.

"A market may be so limited that it is impossible to produce at all and meet the cost of production except by a plant large enough to supply the whole demand". A single producer may be the survivor out of a group of active firms merely by virtue of superior skill, foresight and industry."

Judge Hand: Monopoly was not "thrust" upon ALCOA; ALCOA actively sought to maintain its monopoly through policy of preemptive capacity expansion.

Verdict: For the US. Hand remanded the matter to the trial court for a determination of the remedy. In 1947, Alcoa made the argument that problem had solved itself and no judicial action would be required as there were new entrants into the aluminum market (Reynolds and Kaiser). The district court judge ruled against divestiture in 1950, but the court retained jurisdiction over the case for five years, so that it could look over Alcoa's shoulder and ensure that there was no re-monopolization.

Examples of Regulatory Tools

Regulation of NM under Perfect Information

If regulator knows market demand function and monopolist's cost function, then it can tell the monopolist to choose the price (or output) that is best for society.

1. Welfare maximization (first best) \rightarrow MC pricing, with subsidy.

2. Welfare maximization subject to a zero profitability constraint (second best) \rightarrow AC pricing.

Example 3: (Illustrate the two methods graphically for a bridge.)

Remark:

In theory, first best regulation does not create deadweight loss. But raising subsidy (e.g., taxes) can cause distortions elsewhere in economy. Regulation of NM under Incomplete Information

1. Loeb-Magat Proposal.

Suppose regulator knows market demand. Then regulator can implement the following scheme:

Regulator agrees to pay the monopoly a subsidy CS(p) = consumer surplus at p.

Monopoly's revenue is CS(p) + py(p) = T(y(p)). Thus, it chooses p to

$$\max \pi(p) = T(y(p)) - C(y(p))$$

which maximizes welfare.

Problems:

- Awards all social surplus to monopolist.
- Requires large subsidy.

2. Cost Plus Regulation

Suppose regulator can observe total output, total costs, and total revenue but not the demand or cost function.

Regulator imposes the following constraint on monopolist: it is allowed to earn \$s of profit per unit of output.

Under this scheme, monopolist problem is:

$$\max_{y} \pi(y) \text{ subject to } \pi(y) = yP(y) - C(y) \le sy$$

Solution: choose largest y such that

$$s=P(y)-AC(y)$$

Remark: if s is small, regulated monopolist produces more than unregulated monopolist.

Example 4: $C(y) = y^2$, P(y) = 20 - y, s = 1> Unregulated: $y^M = 5$, $P^M = 15$, $\pi^M = 50$.

• Regulated: $y = 9.5, P = 10.5, \pi = 9.5$.

Problems:

- Monopolist has an incentive to pad costs owner can add "costs" in form of reduced effort or perks.
- No incentive to find innovative ways to reduce costs.
- If marginal costs are increasing in y, it can lead to excessive output. (Illustrate by comparing to perfect competition equilibrium)
- 3. Rate of Return Regulation

Most common form of regulation is to tie the restriction on profits to capital stock rather than output.

K = capital stock, L = labor,

r = cost of capital, w = wage rate.

Let *s* denote the mandated rate of return on capital.

Under regulation, monopolist's problem is to choose L and K to maximize sK subject to

(i)
$$y = F(L, K)$$

(ii) $yP(y) - rK - wL \le sK$.

Example 6: $P(y) = 20 - y; y = \min[K, L/3]; r = w = 1.$ $\blacktriangleright C(y) = 4y.$

• Unregulated:
$$y^M = 8, P^M = 12, \pi^M = 64.$$

Suppose monopolist can keep only \$0.20 for each \$1 of capital.

Thus, if y = 8, then K = 8, and its profits are \$1.60.

To maximize profits, monopolist can expand its capital stock and hence production. What's the best that it can do?

Note that the production function implies that one unit of output requires one unit of capital and three units of labor so we can write the constraint in terms of output:

$$.20y = y(20-y) - 4y$$

Solving for y yields

 $y = 15.8, P = 4.2, \pi =$ \$3.16

Problems:

 Distorts monopolist's choice of technology towards more costly, capital intensive technologies.

 * We know it's more costly/ inefficient because the monopolist didn't choose it to begin with

May lead to excessive output if marginal costs are increasing.

Regulation of Multi-product Firms

Firm produces two outputs, x and y.

Definition: A multi-product industry is a Natural monopoly with respect to quantities x and y if production of x and y by one firm is less expensive than their production by any combination of firms.

Example 7: C(x, y) = F + cx + dy.

Definition: A NM displays Economies of Scope. These are present if C(x, y) < C(x, 0) + C(0, y).

Assumption: demands for x and y are independent. This implies

$$T(x,y) = T_x(x) + T_y(y)$$

where T_x is area under demand curve for x and T_y is area under demand curve for y.

How to regulate?

First best: MC pricing; $p = MC_x$, $q = MC_y$, coupled with subsidy if necessary.

Example 7 (cont.): p = c, q = d, subsidy = F.

Second best: pricing subject to nonnegative profit constraint.

Definition: Ramsey prices are prices p_R and q_R which maximize

 $T_x(x(p)) + T_y(y(q)) - C(x(p), y(q))$

subject to

$$px(p) + qy(q) \ge C(x(p), y(q))$$

Example 8: C(x, y) = 5 + x + y; C(x, 0) = 4 + x; C(0, y) = 2 + y

$$P = 4 - x$$
; $y(q) = 3$ if $q \le 4, 0$ otherwise.

First-best: p = 1 and $q \in [0, 4]$.

We know the first-best prices maximize welfare but we need to check if these actually satisfy the budget constraint. Under the first-best, the firms sets $p_R = 1$ then the budget constraint implies

$$3 + 3q_R = 5 + 3 + 3$$
$$\Rightarrow q_R = \frac{8}{3}$$

Solution: $p_R = 1, q_R = 8/3$

*NB, this is still a first-best allocation since $q_R \in [0, 4]$.

Example 9: same costs as above but

P = 4 - x; y(q) = 3 if $q \le 2, 0$ otherwise.

Solution: $p_R = 2, q_R = 2 \rightarrow$ some deadweight loss in market for x. See lecture notes for more detail.

Intuition: to minimize deadweight loss, tax the product with inelastic demand more heavily.

Definition: Assume the regulated monopoly sells quantities x and y at prices p and q such that its profits are zero. If px > C(x,0) or qy > C(0,y), then the regulated prices p and q involve cross subsidy.

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(See example in notes.)
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Cross subsidy poses problems for the regulator:

1. An entrant may enter the subsidizing market even though efficiency dictates joint production.

2. Consumers of the subsidizing good may object.

Examples: long distance vs. local telephone service; urban vs. rural transportation; large vs. small cities in airline markets.

Possible solution: add constraints on p and q so that they do not involve any cross subsidies.

Problem: no solution may exist!

Key Statutes

Key Statutes - Sherman Act (1890)

- Outlaws "every monopolization, attempted monopolization, or conspiracy or combination to monopolize."
- Outlaws "contract, combination, or conspiracy in restraintof trade."
- Supreme Court decided that it does not prohibit every restraint of trade, only those that are unreasonable (ie, have big effects).

Key Statutes - Clayton Act (1914)

- Amends Sherman Act to address specific practices.
- Prohibits mergers and acquisitions where the effect "may be substantially to lessen competition, or to tend to create a monopoly."
- Bans certain discriminatory prices, services, and allowances in dealings between merchants (Robinson-Patman, 1936).
- Firms required to notify gov't of large mergers or acquisitions (Hart-Scott-Rodino, 1976).

Key Statutes - Federal Trade Commission Act (1914)

- Established the Federal Trade Commission (FTC) a government entity to regulate questionable business practices.
- The FTC is empowered to
 - prevent unfair methods of competition, and unfair or deceptive acts or practices in or affecting commerce;
 - · seek monetary redress and other relief for conduct injurious to consumers;
 - prescribe trade regulation rules defining with specificity acts or practices that are unfair or deceptive, and establishing requirements designed to prevent such acts or practices;
 - conduct investigations relating to the organization, business, practices, and management of entities engaged in commerce; and
 - make reports and legislative recommendations to Congress.