Strategic Pricing

## Strategic Pricing

We now turn to investigate how a firm with market power behaves when faced with heterogenous consumers and products.

1. Price Discrimination
2. Bundling \& Tying
3. The Multi-Product Monopolist
4. Two-Sided Markets

We restrict our attention to a single firm which solves a static optimization problem. Later in the semester we'll add strategic behavior (oligopoly) and dynamic optimization.

1. Price Discrimination

## Price Discrimination

Single-price monopolist leaves some gains from trade on the table. Objectives of this lecture are:

- Discuss more sophisticated pricing strategies firms use to obtain these gains.
- Discuss the welfare implications of these strategies.


## Definitions

First degree PD - firm extracts the maximum willingness to pay for each unit from every consumer.
N.b., This concept is a hypothetical benchmark and is not empirically-relevant.

Third degree PD - firm sells to different, distinguishable groups of consumers at different prices.

- Examples: discounts based on age or location.

Second degree PD - firm offers different units at different prices but it cannot exclude consumers from any offer that it makes.

- Examples: quantity discounts, time discounts


## Motivation - What the Law Says

Robinson-Patman Act (1936): modifies Section 2 of the Clayton Act (1914).

- Prohibits a firm from price discriminating if it harms competition among the firm's customers.
- Passed in response to political pressure from small retail stores that complained that large chains were able to purchase supplies on more favorable terms and thereby charge lower prices.


## Utah Pie Company vs Continental Baking Co. et al

The Market: manufacture and sale of frozen dessert pies (apple, cherry, boysenberry, peach, pumpkin, and mince), primarily in Salt Lake City over the period 1958- mid-1961.

Suppliers: Utah Pie Co. (local producer), Continental Baking Company, Carnation Company, and Pet Milk Company.

- Market growth: 57,060 pies in 1958, 111,729 in 1959, 184,569 in 1960 and 266,908 in 1961.
- Utah Pie's market shares in these years: $66.5 \%, 34.3 \%, 45.5 \%$, and $45.3 \%$.
- Utah's only plant is in Salt Lake city; Pet had plants in Michigan, Pennsylvania, and California; Continental in Virginia, Iowa, and California; Carnation in California.
- All suppliers sold primarily on a delivered price basis.
- Suppliers competed primarily in price. Prices of Utah's pies fell from $\$ 4.15$ per dozen at entry to $\$ 2.7544$ months later. Utah had a cost advantage of lower transport costs.

The Charge: Utah accused the other three companies of price discrimination. They charged lower prices in Salt Lake City than in their other markets.

## Evidence:

- Pet viewed Utah as a thorn in their side; they sent a spy into Utah's plant to obtain information that would be useful in convincing Safeway not to carry Utah's pies. Pet admits that it suffered substantial losses in competing with Utah, greater than in other markets.
- Continental prices its 22 ounce frozen applies at $\$ 2.85$ per dozen in Utah and sold its pies at higher prices elsewhere. Salt Lake price was less than direct cost plus an allocation for overhead. (Predatory pricing.)
- Carnation slashed prices in Utah by 60 cents per dozen which brought its prices below cost. Delivered prices were 20-50 cents lower than prices charged in San Francisco.

Questions: Does price discrimination decrease welfare? Should it be banned?

## Theory

## I. First-Degree PD

Definition: A two-part tariff is a set of prices $(E, p)$ where the consumer pays a fixed fee $E$ for the right to purchase the good and then pays price $p$ per unit.

Total cost of purchasing $q$ units is $E+p q$.
Applications: telephone, cell services.

Example: Consider the pricing problem of a jazz club that offers music and drinks. Inverse demand of the representative patron is

$$
P=V-Q
$$

where $P$ is the price of a drink, $Q$ is the number of drinks.
The club incurs a cost of $c$ dollars per drink.
In addition to charging $P$ per drink, the club levies a cover charge $E$.
Q: What are the profit-maximizing values of $E$ and $P$ ?

## Q: What are the profit-maximizing values of $E$ and $P$ ?

The club's maximization problem is to choose $P$ and $E$ to maximize

$$
\max _{E, p} \underbrace{(V-P)}_{Q(P)}(P-c)+E
$$

Instinct might be to start applying calculus to find FOCs but that's not the right approach.

First, note that at any $P$ the patron anticipates buying $V-P$ drinks which generates a consumer surplus of

$$
\operatorname{CS}(P)=1 / 2 \times(V-P)^{2}
$$

This is the consumer's willingness to pay to enter the club (i.e., $T_{k}(P)$ in the earlier lectures) where the consumer will enter the club anytime

$$
C S(P) \geq E(\text { Utility Maximization })
$$

The club knows this and it further knows it can extract all WTP by setting setting $E=C S(P)$ (i.e., profit-maximization). At this point, the consumer is indifferent between entering and not.

Hence, the club's maximization problem is to choose just $P$ to maximize

$$
\begin{aligned}
& \max _{p} \overbrace{(V-P)(P-c)+\underbrace{\frac{(V-P)^{2}}{2}}_{E(P)}}^{\pi(P, E)} \\
& \text { FOC: } V+c-2 P-(V-P)=0
\end{aligned}
$$

Solution: $P^{\star}=c, E^{\star}=\frac{(V-c)^{2}}{2}$.
Conclusion: Club should price drinks at marginal cost and extract the consumer's surplus at that price through the cover charge. Outcome is efficient!

Remark: if the club books a more popular band, consumer's willingness to pay increases and the club optimally responds by increasing the cover charge but not price of drinks.

## II. Third Degree PD

Two conditions necessary:

- Firm can divide its market into separate sub- markets with different patterns of demand.
- Resale between the two markets by consumers (or third parties) is not possible or illegal.

Pricing Rules:

1. When monopolist can charge different prices in different markets:

- $\mathrm{MR}=\mathrm{MC}$ in each market (assuming it is profitable to serve both markets).

Example: Hardcover copy of Harry Potter and Order of Phoenix.

- Costs $=\$ 4$ per book
- Two Markets: U.S. and Europe

$$
\begin{aligned}
& P_{U}=36-4 Q_{U} \\
& P_{E}=24-4 Q_{E}
\end{aligned}
$$

Solve for uniform (i.e., $p^{\star}$ ) and 3DPD prices (i.e., $\left(p_{U}^{\star}, p_{E}^{\star}\right)$ ), quantities, and profits and graph the solutions.
2. When monopolist can use two-part tariffs:

- Price $=$ marginal cost
- Fixed fees $=$ consumer surplus for each consumer group

Example:
Suppose jazz club can identify consumers by age. Inverse demands for drinks for old and young consumers are

$$
\begin{aligned}
& P_{o}=16-Q_{o} \\
& P_{y}=12-Q_{y}
\end{aligned}
$$

Drinks cost $\$ 4$. Then $P=\$ 4, E_{o}=72, E_{y}=32$.
Q. What is the impact of 3rd degree PD on quantity and welfare? Should it be illegal?

For the following examples, set costs equal to zero and focus on the effects of PD on profits through demand and revenue.

Example 1: PD results in lower output and welfare.

$$
y_{1}\left(p_{1}\right)= \begin{cases}0, & \text { if } p_{1}>4 \\
10, & \text { if } 3<p_{1} \leq 4 ; y_{2}\left(p_{2}\right)=\left\{\begin{array}{ll}
0, & \text { if } p_{2}>3 \\
48, & \text { if } 0 \leq p_{2} \leq 3
\end{array} \text { if } 0 \leq p_{1} \leq 3\right.\end{cases}
$$

|  | PD | No PD |
| :--- | :--- | :--- |
| $\pi$ | $\pi(4,3)=10 \times 4+48 \times 3=184$ | $\pi(3,3)=12 \times 3+48 \times 3=180$ |
| $Y$ | $y_{1}=10, \quad y_{2}=48, Y=58$ | $y_{1}=12, \quad y_{2}=48, Y=60$ |
| $C S$ | $0+0=0$ | $1 \times 10+0=10$ |
| $W$ | $184+0=184$ | $180+10=190$ |

## Learning Checkpoint

Example 2: PD results in higher output and welfare.

$$
y_{1}\left(p_{1}\right)=\left\{\begin{array}{ll}
0, & \text { if } p_{1}>4 \\
100, & \text { if } 0<p_{1} \leq 4
\end{array} ; y_{2}\left(p_{2}\right)= \begin{cases}0, & \text { if } p_{2}>3 \\
20, & \text { if } 0 \leq p_{2} \leq 3\end{cases}\right.
$$

## Learning Checkpoint

Example 2: PD results in higher output and welfare.

$$
y_{1}\left(p_{1}\right)=\left\{\begin{array}{ll}
0, & \text { if } p_{1}>4 \\
100, & \text { if } 0<p_{1} \leq 4
\end{array} ; y_{2}\left(p_{2}\right)= \begin{cases}0, & \text { if } p_{2}>3 \\
20, & \text { if } 0 \leq p_{2} \leq 3\end{cases}\right.
$$

|  | PD | No PD |
| :--- | :--- | :--- |
| $\pi$ | $\pi(4,3)=4 \times 100+3 \times 20=460$ | $\pi(4,4)=4 \times 100+0=400$ |
| $Y$ | $y_{1}=100, \quad y_{2}=20, Y=120$ | $y_{1}=100, \quad y_{2}=0, Y=100$ |
| $C S$ | $0+0=0$ | $0+0=0$ |
| $W$ | $460+0=460$ | $400+0=400$ |

Example 3: PD results in higher output and lower welfare.

$$
y_{1}\left(p_{1}\right)= \begin{cases}0, & \text { if } p_{1}>4 \\
10, & \text { if } 2<p_{1} \leq 4 ; y_{2}\left(p_{2}\right)=\left\{\begin{array}{ll}
0, & \text { if } p_{2}>2 \\
2, & \text { if } 0.02<p_{2} \leq 2 \\
19, & \text { if } 0<p_{1} \leq 2
\end{array}, \begin{array}{l}
\text { if } 0<p_{2} \leq 0.02
\end{array} ~\right.\end{cases}
$$

|  | PD | No PD |
| :--- | :--- | :--- |
| $\pi$ | $\pi(4,0.02)=4 \times 10+0.02 \times 201=44.02$ | $\pi(2,2)=2 \times 19+2 \times 2=42$ |
| $Y$ | $y_{1}=10, \quad y_{2}=201, Y=211$ | $y_{1}=19, \quad y_{2}=2, Y=21$ |
| $C S$ | $0+2 \times 1.98=3.96$ | $10 \times 2+0=20$ |
| $W$ | $44.02+3.96=47.98$ | $42+20=62$ |

Remark: Last example highlights a source of inefficiency from PD monopoly. Units sold may not go to the consumers who value them the highest - there are 9 consumers in market one who are not served, even though they value the good more than 199 of the consumers in market two.

## Conclusions

- PD may actually increase total surplus / welfare (though consumer surplus may decrease).
- Conditional on all markets being serviced, a necessary condition for welfare to increase with 3DPD is that quantity sold increases.
(Varian 1985; Aguirre, Cowen, \& Vickers 2010)


## Application: DellaVigna \& Gentzkow (2019)

- We've shown that profits are unambiguously increasing with 3DPD absent some fixed cost of implementation; i.e.,

$$
\pi^{3 \mathrm{dpd}}-F \geq \pi^{\text {uniform }}
$$

- Do we observe 3DPD or uniform pricing in the data?
- DellaVigna \& Gentzkow use Nielsen scanner data for retail chains to evaluate this question.
- Data: Nielsen Retail Scanner (RMS) and Nielsen Consumer Panel (Homescan) data provided by the Kilts Center at the University of Chicago. RMS records the average weekly revenue and quantity sold for over 35,000 stores and roughly four million unique products (UPCs) for 2006 to 2014.
- Focus on chains; i.e., a retail store with multiple locations.


## An Example

(A) Single Chain, Prices of a Single Product in Orange Juice Category


- Figure depicts prices of one orange juice product.
- The rows correspond to the stores in the chain and are sorted by income. Columns reflect time.
- We observe very little price variation across stores (rows) but do observe variation across time (columns).
- Put differently, we observe uniform pricing in the data for this product.


## More Examples

(B) Single Chain, Prices of Products in Five Categories


- Same pattern across many different categories and products.


## More Examples

(B) Single Chain, Prices of Products in Five Categories


- DVG apply some econometrics to show these pictures are representative of the data as a whole.
- BTW, this paper demonstrates the value of a picture to communicate an idea which econometrics will ultimately show is robust.
- Especially relevant point in a world where "Big Data" can make it hard to see the forest from the trees.


## More Examples

(B) Single Chain, Prices of Products in Five Categories


- Results indicate that uniform pricing is common. Why?


## Implications

- DVG conclude that it's expensive to do 3DPD (i.e., F is big) but that's just a residual explanation.
- Note that F being "big" is a statement of F relative to $\pi^{3 \mathrm{dpd}}-\pi^{\text {uniform }}$.
- Maybe firms are doing something else? Maybe the manufacturers are actually choosing retail prices (Retail Price Maintenance)?
- DVG conclude that uniform pricing may
- significantly increase prices paid by poorer households relative to the rich,
- dampen the response of prices to local economic shocks,
- alter the analysis of mergers in antitrust,
- and shift the incidence of intra-national trade costs.


## III. Second Degree PD

Suppose monopolist knows there are two types of consumers but cannot distinguish them.

The challenge is to construct a pricing scheme so that consumers voluntarily sort themselves into groups and pay different prices.

Example: Consider the jazz club with old and young customers. Recall that their demands are

$$
\begin{aligned}
& P=16-Q_{o} \\
& P=12-Q_{y}
\end{aligned}
$$

Cost of a drink is $\$ 4$. Suppose the club offers the following two menus:
A: entry plus 12 drinks for $\$ 88$
B: entry plus 8 drinks for $\$ 64$
Claim: Old customers choose A, and young customers choose B.
Proof: Compute the consumer surplus for each consumer type for each plan.

$$
\begin{aligned}
& C S_{o}(A)=120-88=32 \\
& C S_{o}(B)=96-64=32 \\
& C S_{y}(A)=72-88=-16 \\
& C S_{y}(B)=64-64=0
\end{aligned}
$$

Q.E.D.

Basic idea:

- B extracts all of the surplus from low types
- A extracts all surplus from high types subject to (incentive) constraint that they prefer A to B .

Notice that this price menu looks like a quantity-discount:

- You can buy 8 items at a price of $\$ 8.00$ per unit, or
- You can buy 12 items at a discounted price of $\$ 7.33$ per unit.

This indicates that when you observe a quantity-discount, the firm faces a variety of consumers and some of these consumers have very high demand. If you're not taking advantage of the discount, the firm's use of 2DPD means your CS is being driven to zero.

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This indicates that when you observe a quantity-discount, the firm faces a variety of consumers and some of these consumers have very high demand. If you're not taking advantage of the discount, the firm's use of 2DPD means your CS is being driven to zero.

Q: Does 2DPD increase firm profits relative to uniform pricing?
A: Not sure. The firm has to give up surplus to the "high" types to get the consumer types to separate (i.e., make different decisions). This sacrifice may be big so equilibrium profits under 2DPD could be less than uniform pricing.

## Example: Sorting through bundling with a "bad".

$$
y(p)= \begin{cases}0, & \text { if } p>4 \\ 20, & \text { if } 2<p \leq 4 \\ 50, & \text { if } p \leq 2\end{cases}
$$

Unit demands; 20 high types value the good at $\$ 4,30$ low types at $\$ 2$; costs are zero.

Here $p^{M}=2, \pi^{M}=100$.

Suppose high types value their time at $\$ 3$ per hour and low types value their time at $\$ 1$ per hour.

Firm can offer two menus:
A: $\$ 3.99$ per unit and no waiting time.
B: \$1 per unit and one hour of waiting time.

## Result:

- "High" types choose A,
- "Low" types choose B, and
- Profits increase from 100 to 110 .


## Proof:

Under A, H types earn $\$ 4-\$ 3.99=.01$.
Under B, H types earn $(\$ 4-\$ 1)-\$ 3=0$.
$\rightarrow$ They choose A.

Under A, L types earn $\$ 2-\$ 3.99<0$.
Under $B, L$ types earn $(\$ 2-\$ 1)-\$ 1=0$.
$\rightarrow$ They choose B.
Profits are

$$
\begin{align*}
& \pi \\
\Rightarrow \pi & \approx  \tag{110}\\
\Rightarrow \pi & \approx
\end{align*}
$$

## Q.E.D.

Remark: monopolist's lack of information about types is socially costly: the 30 Low types waste time.

## Real World Applications

- Intel sold 486 chip for $\$ 588$ and 486 SX chip for $\$ 333$; the SX was the 486 chip with co-processor disabled to slow down computation speed.
- IBM sold LaserPrinter E for $\$ 1000$ less than the LaserPrinter; the E contained extra chips to slow down the printing speed.

A monopolist can also use menus of two-part tariffs to subdivide its market into sub-markets.

- Cell phone companies typically offer a flat-rate plan in which $E$ is high and $p=0$, and a metered plan in which $E$ is low and $p>0$. High users choose the first, low users choose the second.


## Welfare Consequences of PD

1. PD may generate the first best, though consumers get zero surplus.
2. Like SP monopoly, PD monopoly has an incentive to produce too little in any of its markets - deadweight loss in each market.
3. Inefficient allocation across markets in the sense that consumers who value the product more highly are left without it while others get it.
4. Sorting costs due to private information.

Back to our Motivating Question: Should PD be banned?
The answer is no. As shown above, sometimes PD enhances welfare and sometimes it is detrimental. The correct answer is: it depends upon the circumstances.

## Back to the Utah Case

## Recall Market Statistics:

- Market growth: 57,060 pies in 1958, 111,729 in 1959, 184,569 in 1960 and 266,908 in 1961
- Utah Pie's market shares in these years: $66.5 \%, 34.3 \%, 45.5 \%$, and $45.3 \%$.
- Utah's only plant is in Salt Lake city; Pet had plants in Michigan, Pennsylvania, and California; Continental in Virginia, Iowa, and California; Carnation in California.
- All suppliers sold primarily on a delivered price basis.
- Suppliers competed primarily in price. Prices of Utah's pies fell from $\$ 4.15$ per dozen at entry to $\$ 2.7544$ months later. Utah had a cost advantage of lower transport costs.


## Supreme Court Ruling

Majority Ruling: Court found for the Utah, arguing that the price discrimination eroded competition by harming Utah. (Note: some evidence of predatory pricing.)

Minority Ruling: The market in 1961 was more competitive than in 1958, so how could the actions of the respondents be interpreted as anti-competitive?

- "Court has fallen into the error of reading the Robinson-Patman Act as protecting competitors, instead of competition."


## An Empirical Example of Price Discrimination: Dube and Misra (2017) "Scalable Price Targeting"

- The growing access to vast customer databases and analytic tools has made the practice of personalized targeted marketing more accessible to the mainstream firm.
- Authors study scalable price targeting (SPT) which is an extreme form of 3DPD that targets prices using large quantities of observable customer features.
- Theorists have long recognized the possibility that with a very granular segmentation scheme, like SPT, 3DPD could approximate first-degree, or "perfect," price discrimination.


## Council of Economic Advisers (CEA)

- In 2015, publishes a report on the impact of differential pricing enabled by Big Data.
- CEA argued that "...big data and electronic commerce have reduced the costs of targeting and first-degree price discrimination" (CEA, 2015, page 12).
- CEA concluded:
[Differential pricing] transfers value from consumers to shareholders, which generally leads to an increase in inequality and can therefore be inefficient from a utilitarian standpoint (CEA, 2015, page 6)

Research Question: What is the impact of SPT on consumer welfare and firm profits?

## Empirical Setting

- Partnered with Ziprecruiter.com, and online job-search platform, to conduct a series of of randomized controlled price experiments.
- Customers are firms looking to fill vacancies.
- Descriptive evidence that the current $\$ 99$ / month fee was on the inelastic portion of demand so the firm could increase profits by increasing price.


## Experiment 1 (Phase 2)

- Potential new customers (usually HR department of a small firm) fill-out a questionnaire prior to knowing the price of placing a job ad.
- Each potential new customer $(7,867)$ then is given a randomized price from $\$ 19$ to $\$ 399$.
- The customer decides whether to accept (buy) or decline (not buy).


## Estimated Demand



## Estimated Demand, cont'd

- Estimate price elasticity of -0.36 so increasing price will increase profits (inverse elasticity rule).
- Optimal uniform price is $\$ 280.54$.
- Estimate a lot of variation in consumer WTP $\Rightarrow$ uniform pricing doesn't maximize profits.
- Ziprecruiter.com increased profits $68 \%$ by increasing its uniform price from $\$ 99$ to $\$ 249$. Note: the company thought $\$ 280$ was too high.


## Estimated Demand, cont'd

Panel (a): Price Coefficient


Panel (b): Customer Surplus


## Scalable Pricing Targeting (Phase 3)

- Implement SPT using:

1. Estimated demand from Phase 2, including systematic variation between consumer characteristics and WTP.
2. Big Data algorithm (LASSO) to identify most-likely WTP of each consumer based on the characteristics they offer.

- Find that:

1. SPT pricing increased profits $10 \%$ relative to the optimal uniform price.
2. Aggregate CS decreases though the amount is small $(<1 \%)$.
3. Most consumers ( $67 \%$ ) actually paid less under SPT than under the optimal uniform price.

## 2. Bundling and Tying

## Bundling and Tying

## Definitions:

A pure bundling strategy is when the monopolist offers to sell units of two products, 1 and 2, only as a bundle at a price $P_{12}$.
A mixed bundling strategy is when the monopolist offers to sell a unit of product 1 alone at price P1, a unit of product 2 alone at price $P_{2}$, and units of both products at price $P_{12}$.
Stigler Example: One distributor, two films (1 and 2), and two television stations ( A and B ).

|  | Maximum Will- <br> ingness to Pay <br> for Film 1 | Maximum Will- <br> ingness to Pay <br> for Film 2 |
| :--- | :--- | :--- |
| Station A | $\$ 8,000$ | $\$ 2,500$ |
| Station B | $\$ 7,000$ | $\$ 3,000$ |

Optimal uniform pricing policy: $P_{1}=7,000, P_{2}=2,500$, Revenues $=19,000$.
Optimal Pure Bundling Strategy:
$P_{12}=\$ 10,000$, Revenues $=20,000$.

This example demonstrates how bundling can improve profits.
Remarks:

1. Distributor has to know the stations' willingness to pay.
2. The gains from bundling arise from differences in the buyers' relative valuations: $A$ values film 1 more than $B$, and $B$ values film 2 more than $A$.

- Suppose instead that A valued film 2 at $\$ 3,000$ and $B$ valued it at $\$ 2,500$. Then the optimal bundle price is $\$ 9,500 \equiv$ the sum of the optimal uniform prices.

3. In the example, there is no reason for the distributor to consider a mixed bundling strategy. But this is not always true.

## Motivating Case: <br> Eastman Kodak vs Image Technical Services (1992)

## The Charge:

Kodak unlawfully tied the sale of service for Kodak machines to the sale of parts, violating Section 1 of the Sherman Act.

A tying arrangement is "an agreement by a party to sell one product but only on the condition that the buyer also purchases a different (or tied) product, or at least agree that he will not purchase that product from any other supplier."

Such arrangements violate the Sherman Act if the seller has appreciable economic power in the tied market.

Questions: How can a firm bundle/ tie products to increase profits? What is the effect of bundling / tying on welfare?

## A More General Model of Bundling

Supply:

- Two goods, 1 and 2
- Zero marginal costs.

Demand:

- N heterogenous consumers indexed by $k=1, \ldots, N$.
- Unit demands.
- $R_{1}^{k}=$ a consumer's reservation price for good 1 .
- $R_{2}^{k}=$ a consumer's reservation price for good 2 .
- $R_{12}^{k}=$ consumer's reservation price for the bundle of one unit of each good.
- $C S_{i}^{k}=R_{i}^{k}-P_{i}, i=1,2$.

Additivity Assumption: $R_{12}^{k}=R_{1}^{k}+R_{2}^{k}$.
$R_{1}^{k}$ and $R_{2}^{k}$ are distributed independently (and uniformly) on $[0,1]$

- Probability that a randomly selected consumer will buy a unit of good $i$ at price $p$ is simply the probability that $R_{i}^{k} \geq p=\int_{p}^{1} d R_{i}=1-p$.

With lots of consumers in the market, law of large numbers implies $1-p$ is the fraction of consumers who are willing to buy at price $p$ so.

$$
D_{i}(P) \approx N \times(1-p)
$$

## 1. Uniform Pricing

Monopolist sets $P_{1}$ and $P_{2}$ to equate marginal revenue to marginal cost in each market.

$$
\begin{gathered}
\max _{p_{1}, p_{1}} \underbrace{p_{1} \times\left(1-p_{1}\right) \times N}_{\pi_{1}\left(p_{1}\right)}+\underbrace{p_{2} \times\left(1-p_{2}\right) \times N}_{\pi_{2}\left(p_{2}\right)} \\
\Rightarrow 2 p_{1}-1=0 \\
2 p_{2}-1=0
\end{gathered}
$$

The equilibrium is then

$$
\begin{aligned}
p_{1}^{U} & =p_{2}^{U}=1 / 2 \\
\Rightarrow \pi^{U} & =N \times(1 / 4+1 / 4) \\
\pi^{U} & =\frac{N}{2}
\end{aligned}
$$

## Consumer Choices Under Uniform Pricing



- There are four distinct regions of consumers:
$\Gamma(0,0)=\left\{\left(R_{1}^{k}, R_{2}^{k}\right) \mid R_{1}^{k}<1 / 2, R_{2}^{k}<1 / 2\right\}$
$\Gamma(1,0)=\left\{\left(R_{1}^{k}, R_{2}^{k}\right) \mid R_{1}^{k} \geq 1 / 2, R_{2}^{k}<1 / 2\right\}$
$\Gamma(0,1)=\left\{\left(R_{1}^{k}, R_{2}^{k}\right) \mid R_{1}^{k}<1 / 2, R_{2}^{k} \geq 1 / 2\right\}$
$\Gamma(1,1)=\left\{\left(R_{1}^{k}, R_{2}^{k}\right) \mid R_{1}^{k} \geq 1 / 2, R_{2}^{k} \geq 1 / 2\right\}$


## II. Pure Bundling

The monopolist offers a bundle consisting of one unit of each good at price $P_{12}$ and does not allow consumers to buy the goods separately.

One can show that the profit-maximizing price of the bundle cannot exceed the sum of the monopoly prices: $P_{12} \leq P_{1}^{U}+P_{2}^{U}$.
In our example, consumer utility maximization means that demand is given by

$$
D(P)=N\left(1-P^{2} / 2\right)
$$

so optimal bundle price is $P_{12}=(2 / 3)^{1 / 2} \approx .82$. Demand is $2 / 3$.
The optimal pure bundling strategy partitions the space of consumers into two sets:

$$
\begin{aligned}
\Gamma(0,0) & =\left\{\left(R_{1}^{k}, R_{2}^{k}\right) \mid R_{1}^{k}+R_{2}^{k}<.82\right\} \\
\Gamma(1,1) & =\left\{\left(R_{1}^{k}, R_{2}^{k}\right) \mid R_{1}^{k}+R_{2}^{k} \geq .82\right\}
\end{aligned}
$$

## Consumer Choices Under Pure-Bundling Pricing

The partition is no longer quadrants but rather two sets: one corresponding to the set of consumers who do not buy either good and the other to the set of consumers who buy the bundle (i.e., buy both goods).


- Consumers who buy both goods under the uniform pricing will continue to do so under pure bundling.
- Some consumers who previously did not buy any units now buy a unit of both goods.
- Consumers who bought only one good under uniform pricing but who have relatively low reservation values for both goods will buy none.
- Consumers who bought only one good under uniform pricing but who have a high reservation value for one of the goods will tend to buy both.

Example: cable TV.

## Is Pure-bundling a Profit-Maximizing Strategy?

Monopolist faces a trade-off in choosing between pure bundling and uniform pricing: it loses sales from some customers and gains sales from others.

Which pricing policy is better depends upon marginal costs and distribution of reservation prices in the population.

In uniform case, bundling is more profitable. Profits are
$(2 / 3)(2 / 3)^{1 / 2} \approx .55>1 / 2$.
Remark: Suppose unit costs are positive. Under bundling, the allocation is inefficient for two reasons:

1. Some consumers do not buy any units even though their willingness to pay exceeds costs.
2. Some consumers buy units even though their willingness to pay for one of the goods is less than cost.

## III. Mixed Bundling

Remark: Mixed bundling where stand-alone prices $P_{1}$ and $P_{2}$ exceed the bundle price $P_{12}$ is effectively a pure bundling policy.

- No one will want to buy one good alone when the bundle is cheaper (assuming free disposal).
- Therefore, mixed bundling is always at least as profitable as pure bundling and it can be more profitable.

Assume $P_{1}=P_{2}=1 / 2, P_{12}=.82$
Mixed bundling partitions consumers into four sets defined as follows:

$$
\begin{aligned}
\Gamma(0,0) & =\left\{\left(R_{1}^{k}, R_{2}^{k}\right) \mid R_{1}^{k}-1 / 2<0, R_{2}^{k}-1 / 2<0, R_{1}^{k}+R_{2}^{k}<.82\right\} \\
\Gamma(1,0) & =\left\{\left(R_{1}^{k}, R_{2}^{k}\right) \mid R_{1}^{k}-1 / 2 \geq \max \left\{0, R_{1}^{k}+R_{2}^{k}-.82\right\}\right. \\
\Gamma(0,1) & =\left\{\left(R_{1}^{k}, R_{2}^{k}\right) \mid R_{2}^{k}-1 / 2 \geq \max \left\{0, R_{1}^{k}+R_{2}^{k}-.82\right\}\right\} \\
\Gamma(1,1) & =\left\{\left(R_{1}^{k}, R_{2}^{k}\right) \mid R_{1}^{k}+R_{2}^{k}-.82>\max \left\{0, R_{1}^{k}-1 / 2, R_{2}^{k}-1 / 2\right\}\right\}
\end{aligned}
$$

## Consumer Choices Under Mixed-Bundling Pricing



Mixed-bundling provides the monopolist with added flexibility to serve more consumers.

- Some consumers who were not willing to buy the bundle will buy a unit of one of the goods.
- Some consumers who were buying the bundle will switch to buying a unit of only one of the goods.

Derivation:

- $R_{2}=1 / 2$ intersects $R_{2}=.82-R_{1}$ at $R_{1}=.32$.
- $R_{1}=1 / 2$ intersects $R_{2}=.82-R_{1}$ at $R_{2}=.32$.

Examples: music albums, airlines, insurance.

## Tying

Definition: Tying exists when a seller of a product requires as a condition of sale that the customer also purchase a second product (the tied product).

- Main difference with bundling is that tie-in sales do not pre-specify the amounts of each good to be purchased: only one unit of the tied product must be purchased in order for the buyer to purchase the other product.
Example: good $1=$ camera; good $2=$ film.
Two types of consumers: High and Low

$$
\begin{aligned}
& H: Q=16-P \\
& L: Q=12-P
\end{aligned}
$$

where $N_{H}=1$ and $N_{L}=1$.
P is the price of developing a picture. Camera market is a monopoly. Film is produced in a competitive market at a marginal cost of $\$ 2$ per photo.

## A. Monopoly + Competitive Film Market:

- $\mathrm{P}=\$ 2$ per photo.
- H type consumer will take 14 pictures and is willing to pay up to $\$ 98$ (total surplus) to "rent" the camera.
- L type consumer will take 10 pictures and is willing to pay $\$ 50$ (total surplus) to rent the camera.

Thus, the best the monopolist can do is rent the camera at $\$ 50$ and make profits of $\$ 100$.

## B. The Tying Monopolist

Monopolist enters the film market and redesigns the camera so that it can only be used with its film. Costs are $\$ 2$ per photo.

- For now, let $\mathrm{P}=\$ 4$ per photo (we'll show this is $\pi$-maximizing later).

This means

- H types take 12 pictures with $\$ 72$ in consumer surplus
- L types take 8 pictures with $\$ 32$ in consumer surplus.

Thus, the monopolist can rent the cameras for $\$ 32$ and make $\$ 40$ of profit on film for total profits of $\$ 104$.

## Optimal Price of Film

We can now show why setting $p=\$ 4$ maximizes profits. Conditional on selling to both groups (and choosing film price below 12), the monopolist solves:

$$
\begin{array}{r}
\max _{p}[\overbrace{2 \times \underbrace{\frac{(12-p)^{2}}{2}}_{C S^{L}(p)}}^{\text {profit from cameras }}+\underbrace{\text { profit from the film market }}_{\underbrace{(p-2)}_{p-c} \times(\underbrace{16-p}_{y^{H}(p)}+\underbrace{12-p)}_{y^{L}(p)}}] \\
\Rightarrow \frac{d \pi}{d p} \equiv 2 \times(12-p) \times(-1)+(28-2 p)+(p-2) \times(-2)=0 \\
\Rightarrow p=4 \\
\Rightarrow \pi^{T M}=104
\end{array}
$$

If the firm just sells to one group,

$$
\begin{array}{r}
\max _{p}[\underbrace{\frac{(16-p)^{2}}{2}}_{C S^{H}(p)}+\underbrace{(p-2)}_{p-c} \times(\underbrace{16-p)}_{y^{H}(p)}] \\
\Rightarrow \frac{d \pi}{d p} \equiv(16-p) \times(-1)+(16-p)+(p-2) \times(-1)=0 \\
\Rightarrow p=2 \\
\Rightarrow \pi^{T M}=98
\end{array}
$$

Which is the exact same solution as in the jazz club example when the firm creates a two-part tariff.

Equilibrium: The monopolist sets the camera price equal to $\$ 32$, the film price equal to $\$ 4$, and earns $\pi^{T M}=104$.

## Comments:

- When there are heterogenous consumers and the monopolist uses a two-part tariff to sell to multiple consumer types, unit price exceeds marginal cost.
- Monopolist improves profits but fails to extract all consumer surplus. Here, H type consumers have positive surplus.


## C. PD Monopolist

The best strategy is for the monopolist is to design two cameras:

- Camera A can take 14 pictures
- Camera B can take 10 pictures

Consumer just pays for the camera (i.e., he/she pays nothing for developing film).
The monopolist still incurs a marginal cost of $\$ 2$ to develop each photo.

Claim: Monopolist will charge $\$ 86$ for Camera A and $\$ 70$ for camera B. Check:

Willingness to pay of $L$ types for 10 pictures is

$$
(10)(2)+(10)(10)(1 / 2)=70
$$

Willingness to pay of H types for 10 pictures is

$$
(10)(6)+(10)(10)(1 / 2)=110
$$

Willingness to pay of H types for 14 pictures is

$$
(14)(2)+(14)(14)(1 / 2)=126
$$

Therefore, if monopolist charges $\$ 70$ for camera with 10 pictures, the most that it can charge the H types for the camera with 14 pictures is

$$
110-70=126-R, R^{\star}=\$ 86
$$

Total profit is $\pi^{P D}=86+70-2 \times(14+10)=\$ 108>\$ 104=\pi^{T M}$.

## Comments

Remark 1: Camera prices are determined to get the different consumer types to make different decisions. 2DPD may not be profit-maximizing so you next need to check whether the firm would want to choose 2DPD.

Remark 2: The monopolist extracts $L$ type's consumer surplus but H type gets some consumer surplus (CS $=\$ 40$ ).

Question: Does welfare fall when the firm ties the goods?

## 3. The Multi-Product Monopolist

## The Multi-Product Monopolist

Suppose a firm operates two differentiated goods in a single market. What are the optimal prices it chooses? How do these prices vary with changes to cost and demand?

Theory: Two differentiated goods where each good $i=1,2$ has constant marginal cost $c_{i}$. Consumer demands are given by

$$
\begin{aligned}
& y_{1}=A_{1}-p_{1}+s \times p_{2} \\
& y_{2}=A_{2}-p_{2}+s \times p_{1}
\end{aligned}
$$

When $s \in(0,1)$ the goods are substitutes. When $s \in(-1,0)$ the goods are complements.
N.b., I'm imposing that $|s|<1$ so the price effect of another product has less effect on demand than a change in the product's own price.

## Optimization

The Firm chooses $p_{1}$ and $p_{2}$ to maximize profits from both products:

$$
\max _{p_{1}, p_{2}}\{\underbrace{\left(p_{1}-c_{1}\right) \times \overbrace{\left(A_{1}-p_{1}+s p_{2}\right)}^{y_{1}\left(p_{1}, p_{2}\right)}}_{\pi_{1}\left(p_{1}, p_{2}\right)}+\underbrace{\left(p_{2}-c_{2}\right) \times \overbrace{\left(A_{2}-p_{2}+s p_{1}\right)}^{y_{2}\left(p_{1}, p_{2}\right)}\}}_{\pi_{2}\left(p_{1}, p_{2}\right)}
$$

Differentiating,

$$
\begin{aligned}
& \frac{\partial \pi}{\partial p_{1}}=0 \Rightarrow \underbrace{A_{1}-2 p_{1}+s p_{2}+c_{1}}_{\frac{\partial \pi_{1}}{\partial p_{1}}}+\underbrace{s p_{2}-s c_{2}}_{\frac{\partial \pi_{2}}{\partial p_{1}}}=0 \\
& \frac{\partial \pi}{\partial p_{2}}=0 \Rightarrow \underbrace{A_{2}-2 p_{2}+s p_{1}+c_{2}}_{\frac{\partial \pi_{2}}{\partial p_{2}}}+\underbrace{s p_{1}-s c_{1}}_{\frac{\partial \pi_{1}}{\partial p_{2}}}=0
\end{aligned}
$$

## Optimization, cont'd

Simplifying,

$$
\begin{aligned}
& A_{1}-2 p_{1}+c_{1}+s \times\left(2 p_{2}-c_{2}\right)=0 \\
& A_{2}-2 p_{2}+c_{2}+s \times\left(2 p_{1}-c_{1}\right)=0
\end{aligned}
$$

## Comments

- In comparison to a single-product firm, the multi-product firm sets each product's price accounting for the effect on the demand of other products. N.b., we can generate the single-product case by setting $s=0$.
- Moreover, changes in consumer demand (via $A_{i}$ ) and marginal cost (via $c_{i}$ ) for product $i$ affect the pricing decision for all other products (i.e., $-i$ ).
- The degree to which consumer demand and cost for one product spills-over to affect pricing in other products is modulated by "s" which measures the substitutability between the goods, or, equivalently, consumer preferences.
- Here, "s" is a feature of consumer demand but we'll see later in the semester that firms can choose "s" by introducing goods which are more or less similar. Consumer preferences will still be fixed - it's just that firms will optimally choose to introduce products which have characteristics which are not too close.


## How Do Equilibrium Prices Change with s?

- Assume the same linear demand curves as before.
- Let's compare equilibrium prices of a multi-product firm when product demands are unrelated $(s=0)$ and when they're related $(s \neq 0)$.
- Define ( $p_{1}^{\star}, p_{2}^{\star}$ ) as the equilibrium prices when product demands are unrelated ( $s=0$ ) e.g., jet engines and coffee machines.
- Look at the derivatives of the firm profit function (not FONCs) at the point ( $p_{1}^{\star}, p_{2}^{\star}$ ):

$$
\begin{aligned}
& \underbrace{\frac{\partial \pi_{1}\left(p_{1}^{\star}, p_{2}^{\star} \mid s=0\right)}{\partial p_{1}}=0} A_{1}-2 p_{1}^{\star}+c_{1}
\end{aligned} s \times \underbrace{\left(2 p_{2}^{\star}-c_{2}\right)}_{>0}, \underbrace{\underbrace{A_{2}-2 p_{2}^{\star}+c_{2}}_{\frac{\partial \pi_{2}\left(p_{1}^{\star}, p_{2}^{\star} \mid s=0\right)}{\partial p_{2}}=0}+s \times \underbrace{\left(2 p_{1}^{\star}-c_{1}\right)}}_{>0}
$$

- Define $\left(p_{1}^{\prime}, p_{2}^{\prime}\right)$ as the equilibrium prices when product demands are related $(s \neq 0)$. These prices solve the FONCs (i.e., set derivatives equal to zero). What is the relationship between ( $p_{1}^{\prime}, p_{2}^{\prime}$ ) and ( $p_{1}^{\star}, p_{2}^{\star}$ )?
- When goods are substitutes, $s \in(0,1)$ so the above derivative is positive:
$\rightarrow$ Equilibrium prices increase in order to satisfy FONCs $\left(p_{1}^{\prime}>p_{1}^{\star}\right)$.
- When goods are complements, $s \in(-1,0)$ so the above derivative is negative:
$\rightarrow$ Equilibrium prices decrease in order to satisfy FONCs $\left(p_{1}^{\prime}<p_{1}^{\star}\right)$.


## 4. Two-Sided Markets

## Two-Sided Markets (aka "Platforms")

- Thus far we've considered markets where a firm produces a good or service and sells it directly to a consumer.
- There exist other markets, however, where the firm sits between two different sets of consumers. The product the firm offers is a market by which these two consumer groups exchange goods and services.
- For example, newspapers sell content to consumers while also selling advertising to firms. Television, radio, and social media platforms (e.g., facebook) act similarly but set the price of content equal to zero.
- Firms that provide platforms create a market to internalize externalities between the two sides of the market: facebook provides a medium to facilitate the transfer of goods, services, and information between consumers and firms.
- Because profitable operation depends on both sides of the market, we will see that the insights and intuition we've developed studying imperfectly competitive one-sided markets doesn't hold for platforms.


## A Model for Credit Card Payments (Rochet \& Tirole, 2003)

- Monopoly supplier of credit cards (i.e., a platform such as Visa).
- Consumers use credit cards to purchase goods and services at local stores.
- Monopolist charges consumers (buyers) $p^{B}$ and merchants (sellers) $p^{S}$ each time a credit card is used at the point-of-sale (POS).
- Cost per transaction equal to c for the platform.
- Define the following "quasi-demand" functions: ${ }^{1}$

$$
\begin{aligned}
& N^{B}=D^{B}\left(p^{B}\right) \\
& N^{S}=D^{S}\left(p^{S}\right)
\end{aligned}
$$

where $N^{B}$ and $N^{S}$ are the number of consumers (buyers) and merchants (sellers), respectively. Assume both functions are downward-sloping.

[^0]
## Optimization

- Total number of credit card transactions is then

$$
D^{B}\left(p^{B}\right) \times D^{S}\left(p^{S}\right)
$$

- The credit card company solves

$$
\max _{p^{B}, p^{S}}(\underbrace{p^{B}+p^{S}}_{\text {Total Fees }}-c) D^{B}\left(p^{B}\right) D^{S}\left(p^{S}\right)
$$

- Two FONCs:

$$
\begin{aligned}
& \left(p^{B}+p^{S}-c\right) \frac{d D^{B}\left(p^{B}\right)}{d p^{B}}+D^{B}\left(p^{B}\right)=0 \\
& \left(p^{B}+p^{S}-c\right) \frac{d D^{S}\left(p^{S}\right)}{d p^{S}}+D^{S}\left(p^{S}\right)=0
\end{aligned}
$$

## Optimization, cont'd

- We can rewrite the FONCs in terms of Lerner indices (LHS) and quasi-demand elasticities (RHS):

$$
\begin{aligned}
& \frac{p^{B}-\overbrace{\left(c-p^{S}\right)}^{\text {Marg. Cost }}}{p^{B}}=\frac{1}{\varepsilon^{B}} \\
& \frac{p^{S}-\left(c-p^{B}\right)}{p^{S}}=\frac{1}{\varepsilon^{S}}
\end{aligned}
$$

where $\varepsilon^{B}$ and $\varepsilon^{S}$ are the price elasticities of the buyer and seller quasi-demand functions, respectively.

- Comparative Statics:

1. As the firm decreases $p^{B}$, more buyers are willing to carry a credit card (downward-sloping demand) and therefore there are more transactions and ultimately more costs. This is the standard one-sided effect.
2. In a two-sided market, the firm also gains fee revenue $p^{S}$ from the seller so the incremental cost is smaller by $p^{S}$, or equivalently the marginal cost is $c-p^{S}$.
3. Similar logic holds for a decrease in $p^{S}$.

## An Inverse Elasticity Rule for Two-Sided Markets

- We can combine the FONCS to get

$$
\frac{\overbrace{p^{B}+p^{S}}^{\text {Total Price }}-c}{\underbrace{p^{B}+p^{S}}_{\text {Total Price }}}=\frac{1}{\varepsilon^{B}+\varepsilon^{S}}
$$

or equivalently

$$
\frac{p^{B}}{p^{S}}=\frac{\varepsilon^{B}}{\varepsilon^{S}}
$$

- Thus, the monopolist platform will charge a relatively lower price on the side of the market with the less elastic quasi-demand. This behavior increases the number of transactees to that side of the market thereby increasing the attractiveness of the platform for the other side of the market.


## Comments

1. The platform sets prices to balance quantity demanded and supplied to the mutual benefit of both sides. This is the sense in which the platform internalizes the externality between the buyers and suppliers.
2. One side of the market may face a price which is zero or even negative (e.g., reward points, cash-back).
3. The welfare implications (and therefore the regulation) of platform markets are markedly different than the one-sided markets we usually think about.

- (Fairness) It may be optimal for a dating website to charge men and women different prices, a bar to charge men and women different prices for admission and/or drinks, etc.
- (Cross-subsidies) If $p^{B}<p^{S}$, one might conclude that buyers are receiving a subsidy from sellers but this ignores the fact that sellers benefit from the market existing.
- (Market Power) Measuring market power requires accounting for all prices charged (and costs incurred) by the platform.
- (Predatory Pricing) Charging below marginal cost is perfectly reasonable and perhaps welfare improving. In one-sided markets, predatory pricing has traditionally been thought of as a strategy to push the competition out of the market in order to gain market power and raise prices in the future.

4. Because platforms are so different from one-sided markets, a question the DOJ is struggling with is how best to regulate (or not regulate) social media platforms.

[^0]:    ${ }^{1}$ These are "quasi" demand functions $b / c$ the number of buyers choosing to use a credit card depends upon the price of the good or service, not just the credit card transaction fee.

