Oligopoly Competition in Homogeneous Good Markets

Basic Question

Q: How are prices & quantities determined when a small number of firms compete?

For now, we'll look at markets where firms produce a homogeneous (from the consumers' perspective) product?

Examples: crude oil, grain, lumber, cement, solar (photovoltaic) cells.

Key Issue

Monopoly and competitive firm behavior can be described by an optimization problem: firm chooses quantity to maximize profits given demand or price.

In oligopoly markets, when a firm chooses how much to produce or what price to charge, **its profits depend upon the quantities or prices chosen by its rivals and upon their subsequent reactions to its quantity or price choice.**

Open Questions:

- How to model behavior?
- What is an equilibrium?

Note: Play is inherently dynamic but we will initially consider static models.

Cournot Model

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- Book not successful but was the first to use math to explain economic forces and theories.

- First to write supply and demand as a function of price as well as draw supply and demand curves on a graph;

  i.e., not Alfred Marshall in 1890's "Principles of Economics" as is commonly believed / taught.

Cournot Model

Consider the following *game*:

- Player Set: $i = 1, 2$ (duopoly)
- Strategy for firm i : $y_i \in [0, \infty)$

  *NB, this is the defining feature of Cournot models: Firms choose quantities.[1]
- Payoffs for firm i depend upon not only its output choice (y_i) but also upon the output choice y_j of its rival, firm j:

$$\pi_i(y_i, y_j) = P(y_i + y_j) \times y_i - C_i(y_i)$$

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Interpretation: Two firms simultaneously decided how much to produce for the market. Given these amounts, price adjusts to equate demand to supply.

Makes sense in commodity markets where farmers have to commit to production before prices are determined.

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What is an Equilibrium to this Game?

We have a non-cooperative game between two (or more) strategic agents so we'll use the equilibrium concept developed by John Nash.

Definition: A *Nash equilibrium* is a pair of quantities $(y_1^\star, y_2^\star)$ satisfying

$$\pi_i(y_i^\star, y_j^\star) \geq \pi_i(y_i, y_j^\star)$$

for any $y_i \in [0, \infty)$; $i \neq j$; $i, j = 1, 2$.

Interpretation: each firm is behaving optimally given its conjecture about its rival's choice of quantity and, in equilibrium, their conjectures are correct.

An Example

Suppose two firms

- produce a homogenous good at zero cost,
- sell to consumers with (inverse) demand $P(Y) = 120 - 20Y$, and
- each firm chooses between two outputs: low (1.5) and high (2).

Define payoffs $(\pi_1(y_1, y_2), \pi_2(y_1, y_2))$ then all possibilities are presented in the following table:

Firm 2

Firm 1		$y_2 = 1.5$	$y_2 = 2$
	$y_1 = 1.5$	\$90, \$90	\$75, \$100
	$y_1 = 2$	\$100, \$75	\$80, \$80

The Nash equilibrium is (2, 2).

Notice that this Nash equilibrium is also the "dominant strategy" equilibrium: $y_i = 2$ yields higher payoffs for firm i no matter what firm j chooses.

A More General Example

Environment:

- 2 firms indexed by $i = 1, 2$
- Demand: $P(Y) = a - bY$
- Costs: $C(y_i) = cy_i$; $i = 1, 2$

Recall the definition of a Nash equilibrium is that each firm is doing the best it can given the (expected) move of its opponent:

$$\pi_i(y_i^\star, y_j^\star) \geq \pi_i(y_i, y_j^\star)$$

where the index j refers to other firm (*i.e.*, if $i = 1$ then $j = 2$).

Important: Finding the Nash equilibrium requires figuring out how each firm will reply to the move of its opponent.

The "Best Response" (or "Best Reply") Function

Suppose Firm 1 "believes" that Firm 2 produces y_2.

Its "best response" to that value of y_2 is to choose y_1 to maximizes its profits; *i.e.*, it solves

$$\max_{y_1} \pi_1(y_1, y_2) \equiv \max_{y_1} \Big\{ a - b(y_1 + y_2) - c] \times y_1 \Big\}$$

Differentiating and solving yields Firm 1's "best response" function:

$$y_1 = (a - c - by_2)/2b$$

Similarly, Firm 2's "best response" function is

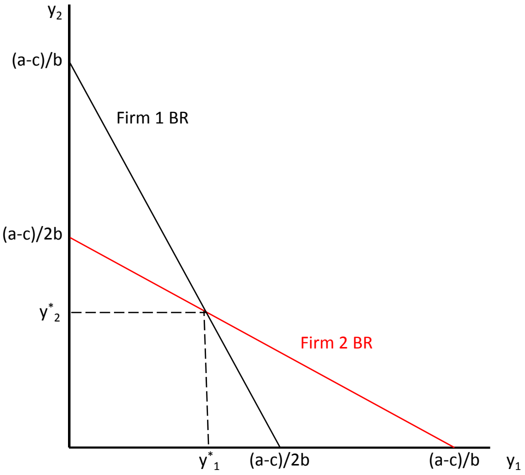
$$y_2 = (a - c - by_1)/2b$$

Note that Firm 1 can't affect / influence the move of Firm 2 (and vice versa) since the firms choose output simultaneously. When we solve for its best response function, we therefore treat Firm 2's move like it's just a number (i.e., $\frac{dy_2}{dy_1} = 0$).

The Nash Equilibrium

The Nash equilibrium is given by the intersection of the two best replies. Note that we can rewrite the best responses s.t. they resemble equations of lines:

$$\mathrm{BR}_1(y_2) = \frac{a-c}{2b} - \frac{y_2}{2}; \;\; \mathrm{BR}_2(y_1) = \frac{a-c}{2b} - \frac{y_1}{2}$$



The Nash Equilibrium

Mathematically, this amounts to solving a simultaneous system of two equations and two unknowns:

$$\begin{cases} y_1 & = \frac{a-c}{2b} - \frac{y_2}{2} \\ \\ y_2 & = \frac{a-c}{2b} - \frac{y_1}{2} \end{cases}$$

The solution is

$$y_1^\star = y_2^\star = (a-c)/3b$$

Thus, equilibrium output, price and firm profits are

$$Y^\star = \frac{2(a-c)}{3b}; P^\star = \frac{a+2c}{3}; \pi^\star = \frac{(a-c)^2}{9b}$$

Remark: If firms reach a non-binding agreement prior to production to produce N.E. quantities, then the agreement is self-enforcing. Each firm cannot gain from deviating if it believes that the other firm will stick to the agreement.

A Comment

Look at Firm 1's problem again:

$$\max_{y_1} \left(a - b(y_1 + y_2)\right) \times y_1 - C_1(y_1)$$

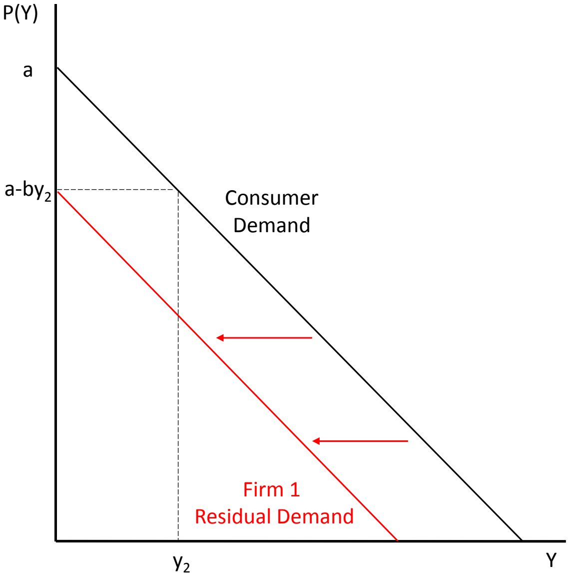
where I'm allowing for a general cost function $C_1(y_1)$ rather than our specific one.

Now move things around a bit:

$$\max_{y_1} \left(\underbrace{a - by_2}_{\substack{\text{Residual}\\ \text{(Inverse)}\\ \text{Demand}}} - by_1\right) \times y_1 - C_1(y_1)$$

So the firm's best reply is the consequence of the firm maximizing profits given the "residual demand" curve which amounts to a downward-shift relative to the original demand curve.

Residual Demand



Extension to N Identical Firms

Let

$$Y_{-i} = \sum_{j \neq i}^{n} y_j$$

denote total output by firm i's rivals. Substituting this quantity into firm i's profit function and differentiating yields its best reply

$$y_i = (a - bY_{-i} - c)/2b$$

Best Responses

In the symmetric equilibrium, firms produce the same amount,

$$y^\star = [a - b(N-1)y^\star - c]/2b$$

Solving yields

$$y^\star = (a-c)/[b(N+1)]$$

Equilibrium output and price are

$$Y^\star = Ny^\star = \frac{N(a-c)}{b(N+1)}; P^\star = \frac{a}{N+1} + \frac{N}{N+1}c$$

Remark: price and output varies from monopoly to perfect competition as N goes from 1 to infinity.

Extension to Asymmetric Firms

Firms always face the same demand curve but their costs functions may vary. In the above example, suppose $C_i(y_i) = c_i y_i$. Let $i = 1, 2$.

Firm i solves

$$\max_{y_i} \left(a - bY_{-i} - by_i\right) \times y_i - c_i y_i$$

Solve for the Best Responses and equilibrium outputs $(y_1^\star, y_2^\star)$.

Extension to Asymmetric Firms (cont'd)

Firm i solves

$$\max_{y_i} \left(a - bY_{-i} - by_i\right) \times y_i - c_i y_i$$

Differentiating, and assuming there are only two firms, the best reply functions are given by

$$y_1 = \frac{(a - c_1 - by_2)}{2b}$$

$$y_2 = \frac{(a - c_2 - by_1)}{2b}$$

Solving the pair of equations yields the N.E.:

$$y_1^\star = \frac{(a + c_2 - 2c_1)}{3b},\ y_2^\star = \frac{(a + c_1 - 2c_2)}{3b}$$

The remaining equilibrium variables (e.g., $P^\star, \pi_1^\star, \pi_2^\star$) follow immediately.

Remarks:

1. In asymmetric case, you need to use all N equations to solve for the N unknowns.

2. Lower cost firm produces more but the higher cost firm produces as long as its costs are not too much higher than the other firm's cost.

The Inverse-elasticity Rule for Cournot Competition

Recall firm i's FOC is

$$[a - bY^{\star}_{-i} - 2by^{\star}_i] - c_i = 0$$

This can be written as

$$P^{\star} - c_i = by^{\star}_i$$

or equivalently,

$$\frac{P^{\star} - c_i}{P^{\star}} = s^{\star}_i \frac{bY^{\star}}{P^{\star}}$$

In the case of linear demand, the demand elasticity is:

$$\eta(P) = \frac{-P}{bY(P)}$$

Thus,

$$\frac{P^{\star} - c_i}{P^{\star}} = \frac{-s^{\star}_i}{\eta(P^{\star})}$$

and we have an inverse-elasticity rule for Cournot with N firms.

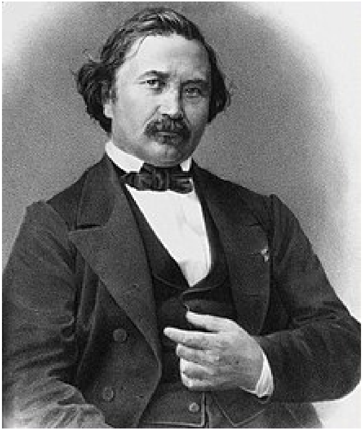
Properties of the Cournot Solution

$$\frac{P^\star - c_i}{P^\star} = \frac{-s_i^\star}{\eta(P^\star)}$$

This is the fundamental markup equation for Cournot markets. Several results follow immediately from this equation:

1. Firms exercise market power since price is above marginal cost $\forall i = 1, ..., N$.
2. As with monopoly, markups are limited by elasticity of demand.
3. All else equal, firms with lower marginal costs have larger market shares.
4. All else equal, market share and markup are both decreasing in the number of firms.
5. If firms are symmetric, the oligopoly Cournot solution lies between the perfectly competitive equilibrium and monopoly in terms of quantities, price, profits, and surplus.

Bertrand Model



- Joseph Louis François Bertrand (1822-1900)
- French mathematician.
- Worked in fields of number theory, differential geometry, probability theory, economics and thermodynamics.
- Developed theory underlying Bertrand model while writing a 1883 review of Cournot's book.
- "Theory" was words / intuition and was not formalized mathematically until Edgeworth in 1889.

Bertrand Model

- Player set: two firms, indexed by $i = 1, 2$.
- Strategy for firm i : $p_i \in [0, \infty)$;
  - * This is the defining feature of Bertrand models: Firms choose price.
- Payoffs for firm i:

$$\pi_i(p_i, p_j) = \begin{cases} (p_i - c_i) \times D(p_i), & \text{if } p_i < p_j \\ (p_i - c_i) \times D(p_i)/2, & \text{if } p_i = p_j \\ 0, & \text{if } p_i > p_j \end{cases}$$

Interpretation: Two firms decide simultaneously what price to charge and produce to demand. Consumers know the prices and buy from the firm that offers the lowest price. Each firm has capacity to serve the entire market.

What is an Equilibrium to this Game?

Definition: A *Nash Equilibrium* is a pair of prices $(p_1^\star, p_2^\star)$ satisfying

$$\pi_i(p_i^\star, p_j^\star) \geq \pi_i(p_i, p_j^\star)$$

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for any $p_i \in [0, \infty); i \neq j; i, j = 1, 2$.

Question: What is the Nash Equilibrium?
Answer: $p_1 = p_2 = c$.

Bertrand paradox: "one is monopoly, two is perfect competition."

Extension to Asymmetric Firms

- Unit costs are $c_1 < c_2 < p^M(c_1)$.
- Assume prices are denominated in pennies.
- Undercutting logic implies the following Nash equilibrium:

$$p_1 = c_2 - .01$$
$$p_2 = c_2$$

- Firm 2 has a cost disadvantage and therefore does not participate (*i.e.*, it exits).

Key Assumptions

1. **Homogeneous Good:** Consumers care only about price and respond en mass to slightest difference in prices.

2. **Static Game:** Firms compete against each other repeatedly in the data.

3. **Unlimited Capacity:** In the data firms generally do not have capacity to service the entire market.

   If all customers try to buy from the low price firm, some customers are rationed and have to buy from the higher price firm - implies prices above marginal cost.

A Bertrand Model with Capacity Constraints

- Two ski resorts: Whistler and Blackcomb.
- Capacity constraints:
  - Blackcomb: 1000 skiers per day
  - Whistler: 1400 skiers per day
- Homoegenous Good: Quality of skiing is same on both mountains.
- Total Skiing Demand: $Q = 6000 - 60P$
- Marginal costs of lift services = \$10 per skier per day.

Important:

If $P = \$10$ (*i.e.*, price equal marg. cost), then total demand is 5400. But supply cannot exceed 2400.

Bertrand Nash Equilibrium with Capacity Constraints

Demand equals total capacity at P = \$60. Suppose both firms charge this price. Would either resort want to deviate?

There are two choices:

1. Decrease price. Clearly, the resort has no incentive to lower price since it cannot serve more skiers. X
2. Increase price. If it raises price above \$60, Whistler faces a residual demand curve:

$$Q_W = 5000 - 60P_W$$

Note: I have assumed that 1000 skiers with highest willingness to pay ski on Blackcomb.

Optimization

Question: Can Whistler Increase Profits by Increasing Price From \$60?

Whistler's optimization problem:

$$\max_{P_W} \pi(P_W) \equiv \max_{P_W} \left\{ (5000 - 60P_w)(P_W - 10) \right\}$$

Differentiating yields

$$\frac{d\pi(P_W)}{dP_W} \equiv 5000 - 60P_W - 60(P_w - 10)$$
$$\equiv 5600 - 120P_W$$

Evaluating at $P_W = \$60$ yields

$$\frac{d\pi(P_W = 60)}{dP_W} = -1600$$

So the derivative is negative which means Whistler's profits fall if it increases price above \$60! X

Whistler's Best Response to $P_b = 60$

- This means that given Blackcomb sets a price of \$60, the best response for Whistler is to set a price of \$60.
- Solving for the best response of Blackcomb yields the same answer, so (60,60) is the Bertrand Nash Equilibrium and equilibrium prices exceed marginal cost.

More Generally

Firms divide the market according to their capacity. But capacity should be viewed as a choice: i.e.,

1. Firms simultaneously choose capacities (y_1, y_2).

2. Given (y_1, y_2) choices in Stage 1, firms simultaneously choose prices (p_1, p_2).

Unfortunately, we're not ready to solve this game, yet. Just wait a couple weeks...

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How much capacity should the two resorts choose when they correctly anticipate the equilibrium prices that will follow?

- Kreps and Scheinkman (1983) show the capacities chosen will be the Cournot quantities!

Thus, the Cournot game is best understood as a capacity game.

Application: Genesove & Mullin (1998) "Testing Static Oligopoly Models: Conduct and Cost in the Sugar Industry, 1890-1914"

- Can we use consumer demand to infer market conduct (e.g., Cournot, Bertrand, Collusion) and firm cost?
- Recall firm FOC under Cournot

$$p + QP'(Q) = c$$

- Modify to include a conduct parameter θ:

$$p + \theta \times QP'(Q) = c$$

where

$$\theta = \begin{cases} 1, & \text{if collusion or monopoly} \\ 0, & \text{if perfect competition or } N \geq 2 \text{ Bertrand} \\ 1/N, & \text{if symmetric Cournot} \end{cases}$$

Empirical Approach

- Rewrite this new FOC as
$$\theta = \eta(P) \times \frac{P - c}{P}$$
- If we know $\{P, c, \eta(P)\}$ (and N), we can infer conduct θ.
- Authors use data on the sugar industry.
  - Reliable estimates of marginal cost
  - Industry underwent significant changes in competition during period.
  - We know firms colluded during certain periods and competed aggressively (price war) during others.
  - Can test whether theory and data are consistent.

Some Results

- Estimate $\{\theta, c\}$ assuming linear demand:

$$\mathrm{E}[\{(1+\theta)P - \alpha\theta - c_o - kP_{RAW}\}\mathbf{Z}] = 0. \tag{12}$$

  where Z are "instruments" for price (more on this later...)

- Suppose we know $\theta = 0.10$ from other data.
- Will our estimates generate a similar value?

Some Results

TABLE 7 NLIV Estimates of Pricing Rule Parameters

	Linear		Direct Measure
	(1)	(2)	(3)
$\hat{\theta}$	.038	.037	.10
	(.024)	(.024)	
$\hat{c}_o$	.466	.39	.26
	(.285)	(.061)	
$\hat{k}$	1.052		1.075
	(.085)		

- $\hat{\theta}$ pretty close to the "Direct Measure". Reject monopoly and Cournot with less than nine firms.
- Can't reject perfect competition $\theta = 0$.
- Costs c imprecisely estimated. Can't reject $k = 1.075$.
- Including information about costs (k) does not help identify θ (column 2).