Oligopoly Competition in Heterogenous Good Markets

Oligopoly Pricing in Differentiated Good Markets

Most markets are differentiated product markets: products of different firms are not perfect substitutes.

Put differently, in most markets consumers make purchase decisions based not only upon price but also product characteristics (*i.e.*, they care about the label on the product).

Our models of these markets are among the most realistic and useful of all models in IO. They provide insights into a range of issues:

- Product placement
- Niche markets
- Product design
- Brand proliferation

BTW I will use the terms "heterogenous" and "differentiated" interchangeably to describe goods where product characteristics other than price matter for consumer purchase decisions.

Two Types of Differentiation

1. **Horizontal Differentiation:** If all products were priced the same, consumers would disagree on which is the most preferred product.

Key attributes on which consumers differ: product characteristics or location

Examples: films, beer, cars, books, cereals, ice cream flavors, Starbucks by geographic location.

Two Types of Differentiation

2. **Vertical Differentiation:** if all products were priced the same, all consumers agree on the preference ranking although they may differ in their willingness to pay.

Key attributes: quality, performance, durability.

Examples: computers, diamonds, batteries.

Modeling Differentiation: Our Plan

In general, a product is a bundle of vertical and horizontal attributes.

- A car is a bundle of certain amount of horsepower (vertical), color, weight, and size (horizontal).

Of course, it takes only one horizontal dimension for products to be horizontally differentiated.

The Plan:

1. We begin by developing a theory for products which are horizontally differentiated in one dimension.
2. For estimation, we want to model consumer demand allowing for different degrees to which products are horizontally vs vertically differentiated.

The Bertrand Paradox Revisited

Does the Choice Variable of the Firms Matter?

- We saw that with oligopoly competition in homogenous goods, whether firms chose quantity (Cournot) or price (Bertrand) mattered for determining the Nash equilibrium (*i.e.*, the "Bertrand Paradox").
- We also showed that adding capacity constraints to the pricing problem lead gave us insight into how to view Cournot competition in the data.
- Absent capacity constraints, we now ask: Does whether firms choose quantities or prices matter when goods are heterogenous / differentiated?

Conceptual Framework - Demand

Consider a market with two differentiated products, 1 and 2.

$$q_1(p_1, p_2) = a - bp_1 + cp_2$$
$$q_2(p_1, p_2) = a - bp_2 + cp_1;\ b > 0, b > |c|$$

Remark: Two goods are substitutes if $c > 0$; they are complements if $c < 0$; also own-price effect, b, has to be larger than cross-price effect c.

Conceptual Framework - Inverse Demand

We can invert the system of demands to obtain inverse demands.

$$p_1(q_1, q_2) = \alpha - \beta q_1 - \gamma q_2$$
$$p_2(q_1, q_2) = \alpha - \beta q_2 - \gamma q_1$$

where

$$a = \frac{\alpha(\beta - \gamma)}{\beta^2 - \gamma^2}, b = \frac{\beta}{\beta^2 - \gamma^2}, c = \frac{\gamma}{\beta^2 - \gamma^2}$$

Pricing Game

Firm 1 chooses price to maximize

$$\pi_1(p_1, p_2) = (a - bp_1 + cp_2)p_1$$

Costs are zero. Differentiating and solving for p_1 as a function of p_2:

$$p_1 = (a + cp_2)/2b$$

Similarly, for firm 2,

$$p_2 = (a + cp_1)/2b$$

Solving for the equilibrium:

$$p_1^\star = p_2^\star = \frac{a}{2b - c}$$

Quantity Game

Firm 1 chooses quantity to maximize

$$\pi_1(q_1, q_2) = (\alpha - \beta q_1 - \gamma q_2)q_1$$

Differentiating and solving for Firm 1's best reply,

$$q_1 = (a - \gamma q_2)/2\beta$$

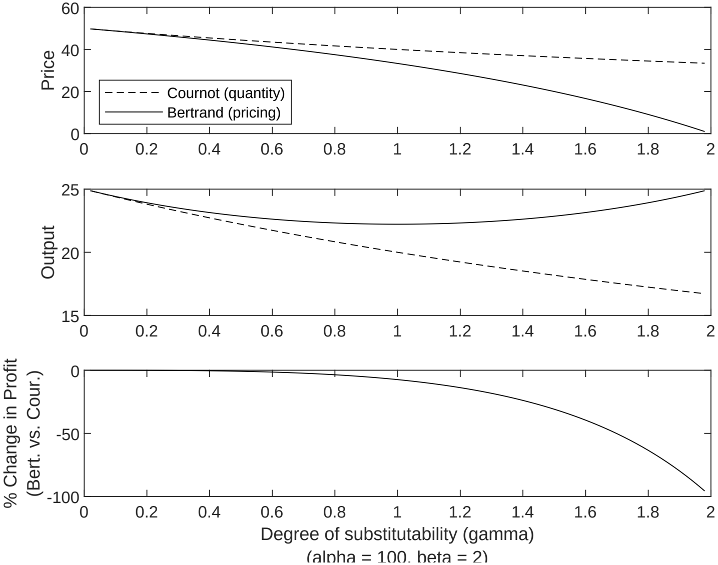
Similarly, Firm 2's best reply is

$$q_2 = (\alpha - \gamma q_1)/2\beta$$

Solving for equilibrium:

$$q_1^\star = q_2^\star = \frac{\alpha}{2\beta + \gamma}$$

Basic Questions: Does how firms interact / compete matter? Specifically, do equilibrium prices and quantities differ in the pricing game versus the quantity game?



Look the best replies in each game:

$$\text{(Pricing) } p_1 = (a + cp_2)/2b$$
$$\text{(Quantity) } q_1 = (a - \gamma q_2)/2\beta$$

Note that good are substitutes when $c > 0$ (& therefore $\gamma > 0$) and just the opposite when goods are complements. Also note that whether or not goods are substitutes or complements difference affects the slope (*i.e.*, the direction of the response) of the best replies.

Definitions: the strategic choices of players are **strategic complements** if best replies slope up; **strategic substitutes** if best replies slope down.

	Goods are:	
	Substitutes	Complements
Firms choose price	strategic complements	strategic substitutes
Firms choose quantity	strategic substitutes	strategic complements

* Note that firm behavior depends on slope of best replies.

Product Differentiation & Regulation

The Turnpike Model

Q: Why do we care if firm choices are strategic complements are substitutes?

A: Because the slope of the best reply functions provides insight into how firms compete as well as the implications of competition.

An Example: *The Turnpike Model*

This model illustrates why antitrust policy in markets with complements is fundamentally different than in markets with substitutes.

- Consider a one mile private road from A to B.
- Players: N individuals (e.g., towns) own segments of the road. Each owner installs a toll booth to collect a toll for her segment.
- Behavior: Owners choose tolls (p_i) simultaneously.
- Demand: $D(P) = a - bP$ where $P = \sum_{i=1} p_i$.
- Costs are zero.

Q: What is the Bertrand Nash equilibrium?

Q: What is the Nash equilibrium?

Owner i chooses p_i to maximize

$$\pi(p_i, p_{-i}) = p_i \times \left(a - bp_i - b\sum_{i \neq j} p_j\right)$$

Differentiating and solving for p_i yields the best response for firm i:

$$p_i = \frac{a - b\sum_{i \neq j} p_j}{2b}$$

We have N best responses which in the Nash equilibrium must hold simultaneously. Symmetry simplifies things significantly, though. We get the following Nash equilibrium:

$$p^\star = \frac{a}{(N+1)b}$$

$$P^\star = \frac{N}{N+1} \times \frac{a}{b}, Q^\star = \frac{a}{N+1}$$

$$\pi^\star = \frac{a^2}{(N+1)^2 b}$$

How Does Competition Affect the Nash Equilibrium?

What happens as N gets large?

- $P^\star$ increases to a/b, $Q^\star$ goes to zero.
- Price is lowest when there is only one firm

Conclusions:

- Consumers are better off with a monopoly than competition!
- Antitrust policy should promote monopoly when goods are complements (*i.e.*, when there are network effects).

Intuition

Think of $Q(P)$ as the size of the pie and $p_i Q(P)$ as individual i's share of the pie.

- Look at the partial derivative of the firm's profit function

$$\frac{\partial \pi_i}{\partial p_i} = \underbrace{Q(P)}_{>0} + \underbrace{p_i \times \partial Q(P)/\partial p_1}_{<0}$$

  which summarizes the firm's motivations:

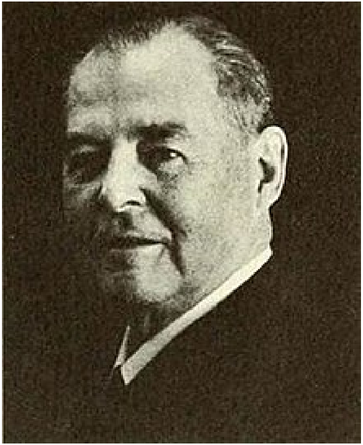
  1. Given a size of the pie $Q(P)$, the firm increases profits by increasing price p_i (first term).
  2. As N grows, the first-order effect of an increase in p_i on the decrease in the size of the pie becomes small (second term).

- From the best response, firm i optimally chooses to increase price when rivals decreases their prices (strategic substitutes); *i.e.*, its best response is to capture a greater share of the pie.
- In the Nash equilibrium, there exists a coordination problem where $p^\star(N)$ is falling too slowly in N b/c the degree to which an individual firm can affect $Q(P)$ gets smaller so the firm internalizes the effects of its price on aggregate demand less and less.
- The interaction of N and $N \times p^\star(N)$ therefore leads equilibrium overall price for the consumer $P^\star(N)$ $\left(i.e., N \times p^\star(N)\right)$ to increases as competition (N) increases.
- What's the Big Picture Idea? In industries where goods are complements (e.g., Networks), battle over higher market shares ends up destroying the pie!

Applications: Apple Store, Google Play Store, Gaming systems, Airlines, Railroads

The Hotelling Model

The Hotelling Location Model of Product Differentiation



- Harold Hotelling (1895-1973)
- Known for theories of "spatial economics"
- Published what is now known as Hotelling's Location Model in "Stability in Competition," published in 1929.
- Famous quote for trivia night or to impress your friends:

  "...a discontinuity, like a vacuum, is abhorred by nature." (p. 44)

The Hotelling Location Model (cont'd)

This model allows us to study product location and variety.

<u>Environment</u>

- One mile long beach; $0 =$ left end point, $1 =$ right end point.
- $M = 1000$ people are distributed uniformly along the beach; thus fraction of bathers in any section of the beach of length z is $zM = 1000z$.
- Location of bather x is measured relative to 0.
- Two ice-cream vendors are located at either end of the beach. Vendors have marginal cost equal to zero.
- Vendors compete in price.
- Each consumer incurs a "Transportation Cost" of t times the squared distance required to walk to the vendor.

Note that the "cost of walking" (i.e., the "transportation cost" of not purchasing one's ideal variety) is our assumption. We could have said it was zero, linear, cubic, etc. Each assumption would deliver a different Nash equilibrium.

Consumer Preferences

- Consumer located at position x has the following utility function:

$$u(x) = \begin{cases} s - p_1 - tx^2, & \text{if x purchase from vendor 1} \\ s - p_2 - t(1-x)^2, & \text{if x purchases from vendor 2} \\ 0, & \text{otherwise} \end{cases}$$

  where s measures the utility for consuming an ice cream cone.

- Total cost of buying from vendor 1 is $p_1 + tx^2$; total cost of buying from vendor 2 is $p_2 + t(1-x)^2$.

- If prices are the same (and s is sufficiently large), then each consumer buys from the vendor who is closer.

Note that we've assumed "s" is the same for both vendors to make things simple. It would not be very hard to allow $s_1 \neq s_2$.
wink, wink: that would be a reasonable exam question!

Case 1: Local Monopoly

Def: "Local Monopoly" occurs when there exists no overlap in the market coverage of the two vendors when they set price equal to the monopoly price.

Q. What is the monopoly price?

- Need to compute demand. Define $\tilde{x}$ as the marginal consumer who is indifferent between buying from vendor 1 and not buying. Mathematically, at $\tilde{x}$ we have

$$
\begin{aligned}
u_1(\tilde{x}) &= 0 \\
\Rightarrow s - p_1 - t\tilde{x}^2 &= 0 \\
\Rightarrow \tilde{x}(p_1) &= \sqrt{(s - p_1)/t}
\end{aligned}
$$

Demand for Vendor 1

- Since all consumers face the same price p, any consumer located at $x \leq \tilde{x}$ gets utility $u_1(x) \geq 0$ from buying from vendor 1 and therefore choose to buy.
- Since x is distributed iid uniform on $[0, 1]$, the probability a randomly chosen consumer will buy good one is

  $$Pr\Big[X < \tilde{x}(p_1)\Big] = \int_0^{\tilde{x}(p_1)} dx = \tilde{x}(p_1)$$

  and by the Law of Large Numbers the number of consumers which buy good one (*i.e.*, demand) is

  $$D_1(p_1) = \tilde{x}(p_1) \times M.$$

Note: "No overlap" means there exists consumers in the middle which do not participate in the equilibrium so $\tilde{x} < 1/2$.

Q. What is the equilibrium monopoly price?

Vendor 1's demand at p_1 is $\tilde{x} \times M$ and it chooses price to maximize

$$\pi(p_1) = p_1 \times \underbrace{(\sqrt{(s - p_1)/t})M}_{D_1(p_1)}$$

Differentiating and solving yields the monopoly price:

$$p_1^M = 2s/3$$

Need to check that $\tilde{x}$ is indeed less than $1/2$ (*i.e.*, no overlap):

$$\tilde{x}(p_1^M) < 1/2 \text{ iff } s/t < 3/4$$

Problem for vendor 2 is symmetric but now demand is $1 - \tilde{y}(p_2^M)$ where $\tilde{y}$ is the location of the consumer indifferent between buying from vendor 2 and not buying at all.

Case 2: Duopoly

Together the two vendors cover the market so the marginal consumer ($\hat{x}$) is indifferent between buying from vendor 1 or vendor 2.

$$\underbrace{p_1 + t\hat{x}^2}_{u_1(\hat{x})} = \underbrace{p_2 + t(1-\hat{x})^2}_{u_2(\hat{x})}$$

$$\Rightarrow \hat{x} = (p_2 - p_1 + t)/2t$$

Demand for vendor 1 decreases in own price and increases in rival's price.

Q. What are the Nash equilibrium prices?

Vendor 1 chooses its price to maximize

$$\pi_1(p_1, p_2) = p_1 \times \underbrace{\hat{x}(p_1, p_2) \times M}_{D_1(p_1, p_2)} = p_1 \times \Big[(p_2 - p_1 + t)/2t\Big] M$$

Differentiating and solving for vendor 1's best reply yields

$$p_1 = (p_2 + t)/2$$

Vendor 2 chooses its price to maximize

$$\pi_2(p_1, p2) = p_2 \times \underbrace{\Big(1 - \hat{x}(p_1, p_2)\Big) \times M}_{D_2(p_1, p_2)} = p_2 \times \Big[(p_1 - p_2 + t)/2t\Big] M$$

Differentiating and solving yields

$$p_2 = (p_1 + t)/2$$

Remark: Prices are strategic complements (*i.e.*, upward-sloping BRs). We therefore know that goods one and two are substitutes.

Q. What are the Nash equilibrium prices? (cont'd)

The intersection of the best replies is

$$p_1^\star = p_2^\star = t$$

Need to check:

1. The market is indeed covered (*i.e.*, $\hat{x} = 1/2$) ✓
2. The marginal consumer gets non-negative utility (*i.e.*, $u_1(\hat{x}) \geq 0$)
   - True iff $\frac{s}{t} > \frac{5}{4}$.

Remark: The parameter t is a measure of how differentiated the products are. If t is zero, then consumers, regardless of location, buy from the vendor offering the lowest price. Prices and profits increase in t.

Learning Check-Point: Different Vendor Locations

Suppose vendor 1 is at $1/4$ and vendor 2 at $3/4$ and market is covered (*i.e.*, competition is a duopoly).

Solve for equilibrium prices.

Solution

Step 1: Determine location of the marginal consumer.

- Marginal consumer is indifferent between buying from vendor 1 or vendor 2.

$$s - p_1 - t(\hat{x} - 1/4)^2 = s - p_2 - t(3/4 - \hat{x})^2$$
$$\Rightarrow \hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{t}$$

Step 2: Derive the profit functions.

- Vendor 1 chooses its price to solve:

$$\max_{p_1} \; p_1 \left(\frac{1}{2} + \frac{p_2 - p_1}{t} \right) M$$

- Vendor 2 chooses its price to solve:

$$\max_{p_2} \; p_2 \left(\frac{1}{2} + \frac{p_1 - p_2}{t} \right) M$$

Note: The firms are solving identical problems $\Rightarrow$ we suspect the best responses will be identical as well.

Solution, cont'd

Step 3: Derive best replies by differentiating and solving the first-order condition.

$$p_1 = \frac{p_2}{2} + \frac{t}{4}$$

$$p_2 = \frac{p_1}{2} + \frac{t}{4}$$

Step 4: Solve for the Nash equilibrium by solving the system of two equations, two unknowns.

Nash Equilibrium: $p_1 = p_2 = t/2$.

Intuition: When vendors are located closer to each other, price competition increases and equilibrium prices fall.

Another Extension: More Vendors

Suppose vendor 1 is at 0 (p_1), vendor 2 is at $1/2$ (p_2), and vendor 3 is at 1 (p_3).

Two marginal consumers:

- $\hat{x}$ is indifferent between buying from vendor 1 or vendor 2
- $\hat{y}$ is indifferent between buying from vendor 2 or vendor 3.

Constructing Demand from Utility-Maximization

Step 1: Determine location of marginal consumers:

$$\begin{aligned} u_1(\hat{x}) &= u_2(\hat{x}) \\ s - p_1 - t\hat{x}^2 &= s - p_2 - t \times (1/2 - \hat{x})^2 \\ \Rightarrow \hat{x} &= \frac{1}{4} + \frac{p_2 - p_1}{t} \end{aligned}$$

$$\begin{aligned} u_2(\hat{y}) &= u_3(\hat{y}) \\ s - p_2 - t \times (\hat{y} - 1/2)^2 &= s - p_3 - t \times (1 - \hat{y})^2 \\ \Rightarrow \hat{y} &= \frac{3}{4} + \frac{p_3 - p_2}{t} \end{aligned}$$

Profit-Maximization

Step 2: Derive profit functions:

- Vendor 1 chooses p_1 to maximize

$$\pi_1(p_1) = p_1 \hat{x}(p_1, p_2) M = p_1 \left(\frac{1}{4} + \frac{p_2 - p_1}{t} \right) M$$

- Vendor 2 chooses p_2 to maximize

$$\pi_2(p_2) = p_2 [\hat{y}(p_2, p_3) - \hat{x}(p_1, p_2)] M = p_2 \left(\frac{1}{2} + \frac{p_3 - p_2}{t} - \frac{p_2 - p_1}{t} \right) M$$

- Vendor 3 chooses p_3 to maximize

$$\pi_3(p_3) = p_3 (1 - \hat{y}(p_3, p_2)) M = p_3 \left(\frac{1}{4} + \frac{p_2 - p_3}{t} \right) M$$

Solving for Nash Equilibrium Prices

Step 3: Derive best replies:

Differentiating and solving first order conditions yields the following system of best replies:

$$p_1 = \frac{p_2}{2} + \frac{t}{8}$$
$$p_2 = \frac{p_3 + p_1}{4} + \frac{t}{8}$$
$$p_3 = \frac{p_2}{2} + \frac{t}{8}$$

Step 4: Solve the system of equations (*i.e.*, find the intersection of the best response functions):

Nash equilibrium prices: $p_1^\star = p_2^\star = p_3^\star = t/4$; $\hat{x}^\star = 1/4$ and $\hat{y}^\star = 3/4$.

Note: Each vendor competes with its neighbor only - direct competition is local. But in the equilibrium, all vendors compete.

Another Extension: Multi-product vendors

Suppose vendor 1 is located at 0 and vendor 2 has carts at locations $1/2$ and 1.

What are the equilibrium prices?

Solution Technique: Solve following the steps we've outlined but now profit maximization for Vendor 2 is

$$\max_{p_{1/2}, p_1} \; \pi_{1/2}(p_0, p_{1/2}, p_1) + \pi_1(p_0, p_{1/2}, p_1)$$

so you have two FOCS and two BRs for Vendor 2 but notice that you have some extra terms compared to the three firm version of the model. Solve the Nash equilibrium by solving the system as before.

Summary

- We can model product differentiation explicitly via estimated demand functions or construct these demand functions from first principles (consumer utility maximization).
- Important Result: Price competition diminishes as product differentiation increases. This is known as the "principle of product differentiation."
- Often the number of firms in an industry is thought of as a sufficient statistic for degree of competition...
- But empirically researchers find market power across a wide variety of settings:
  - In homogeneous product industries, such as cement, transportation costs result in highly localized competition among few firms.
  - In differentiated goods industries, horizontal differentiation between products bestows market power in the part of the product characteristic space where the firm competes.

Question: If product differentiation is a firm choice which drives market power and profits, shouldn't we focus on the strategic choice of which kinds of products are introduced?

A Last Extension: Location Choice

Q: What if firms also chose their location?

Consider the following two-stage game:

1. Vendors simultaneously choose location.
2. Given observed location choices, vendors choose prices simultaneously.

A Last Extension: Location Choice (cont'd)

Two forces affect the location decision:

1. Locate where the demand is - move to the center.
2. Stay away from the competition - go to the ends.

Which force dominates depends upon the "travel" costs.

- If travel costs are quadratic (e.g., tx^2), the second force dominates and firms choose the end-points at Stage One.
- If travel costs are proportional (e.g., tx), the first force dominates and there is not an equilibrium in pure strategies.

Existence

In case you're wondering how changing the transport costs affects the equilibrium pricing and location choices in the Hotelling model...

Paper	Domain	Transport Cost	Pricing Eqm Exist?	Equilibrium Locations
Hotelling (1929)	interval	$d(x) = bx$, $b > 0$	yes	n/a
d'Aspremont, Gabszewicz, & Thisse (1979)	interval	$d(x) = bx^2$, $b > 0$	yes	end-points
Economides (1986)	interval	$d(x) = bx^\alpha$, $b > 0$, $1 \leq \alpha \leq 2$	if $1.26 \leq \alpha \leq 5/3$ if $5/3 \leq \alpha \leq 2$ no otherwise	interior end-points n/a
Gabszewicz & Thisse (1986), Anderson (1988)	interval	$d(x) = ax + bx^2$	no	n/a
Kats (1995)	circular	$d(x) = bx$, $b > 0$	no	n/a
de Frutos, Hamoundi, & Jarque (1999)	circular	$d(x) = ax + bx^2$, $a \geq 0, \;\; b \in \mathbb{R}^1$	if $a = 0$ or $a = -b$ no otherwise	end-points n/a

Estimation

Using Theory to Discipline Data Analysis

- Suppose we have a data set on sales of a differentiated good (e.g., cars in US).
  - prices and quantity sold by model.
  - vehicle characteristics (e.g., horsepower, weight, size, fuel efficiency)
- We have some economic research question in mind, e.g.,

  Q: Do consumers favor national over foreign brands?

  Q: Are subsidies effective at encouraging adoption of a new product?, How much of these subsidies are captured by firms with market power?

  Q: Do consumers make car purchase decisions based on fuel economy?
- We answer these questions by connecting data to theory: We estimate an equilibrium oligopoly model of demand for horizontally-differentiated goods.
- Model needs to
  - capture the most important aspects of the data, but
  - be simple enough that we can solve it.

Theory: A Model of Differentiated Goods

- We're going to extend the ideas embodied in Hotelling.
- The consumer demand model we'll develop is known as "Multinomial Logit."
- It's going to look ugly and complicated at parts but we'll see that the ugliness and complications will disappear with some clever transformations.
  - Daniel McFadden won the 2003 Nobel Prize for developing this model.
  - Often used by the Department of Justice to evaluate anti-trust cases (e.g., collusion, mergers).
- We'll see that estimation will be as simple as doing OLS and IV (2SLS).
- Doing policy analysis is also possible but then we'll have to solve for consumer choices and firm prices (*i.e.*, Bertrand Nash equilibrium prices) conditional on any policy change. This is doable but more complicated.
  - To ignore the ability of consumers and firms to re-optimize their respective decisions is to fall victim of the *Lucas Critique*.

Theory: A Model of Differentiated Goods (cont'd)

- Heterogenous consumers.
- Each can buy one of $J+1$ products.
  - The "Multinomial Logit" model is therefore known as a "discrete choice" model since each consumer buys one unit rather than many units.
  - Note that the Hotelling model is a "discrete choice" model as well.
- Instead of thinking about demand for product $j = 1, ...J+1$ as

$$y_j = a_j - bp_j \ ,$$

recast as demand for product j based on product j's characteristics and price:

$$y_j = \underbrace{\beta_1 X_1 + \beta_2 X_2 + ... + \beta_K X_K}_{a_j} - bp_j \ .$$

Theory: A Model of Differentiated Goods (cont'd)

Each consumer i gets utility u_{ij} from consuming product j:

$$u_{ij} = \underbrace{\beta x_j + \xi_j + \epsilon_{ij}}_{\text{WTP}_{ij}} - \alpha p_j \tag{1}$$

- "x_j" is the characteristics of product j **observed** by the econometrician.
- "ξ_j" is the characteristics of product j **not observed** by the econometrician.
- "p_j" is the price of product j.
- "ϵ_{ij}" is agent i's random taste for product j.

Product "0" is known as the outside option – the value of not buying.

$$u_{i0} = \epsilon_{i0} \tag{2}$$

Theory: Market Shares and Demand

- Consumers are utility maximizers so each consumer solves

$$u_i^\star = \max_j \{u_{ij}\}_{j=0}^J \tag{3}$$

- Assume that the heterogenous tastes (ϵ) come from a GEV distribution.
- We can then solve for the expected probability and agent buys product j:

$$s_j(p) = \frac{\exp\left(\beta x_j - \alpha p_j + \xi_j\right)}{1 + \sum_{k \in J} \exp(\beta x_k - \alpha p_k + \xi_k)}. \tag{4}$$

  where we've normalized $u_{i0} = \epsilon_{i0} = 0$ (similar to Hotelling).
  - Notice the lack of i subscript – all agents have the same probability of buying product j.
  - This $s_j(p)$ is analogous to $\hat{x}(p)$ as the probability of buying from the vendor at the left end-point.
- Define M as the number of people in the market then demand for product j is

$$y_j(p) = s_j(p) \times M. \tag{5}$$

Estimation: Bringing Theory to Data

- Divide (4) by s_0

$$\frac{s_j(p)}{s_0(p)} = \frac{\frac{\exp(\beta x_j - \alpha p_j + \xi_j)}{1+\sum_{k\in J}\exp(\beta x_k - \alpha p_k + \xi_k)}}{\frac{1}{1+\sum_{k\in J}\exp(\beta x_k - \alpha p_k + \xi_k)}}$$

$$\Rightarrow \frac{s_j(p)}{s_0(p)} = \exp\left(\beta x_j - \alpha p_j + \xi_j\right)$$

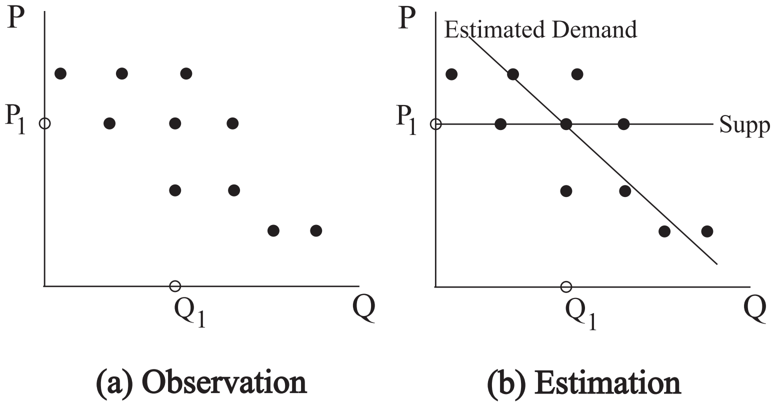
- Apply logs

$$\underbrace{\log(s_j) - \log(s_0)}_{y_j} = \beta x_j - \alpha p_j + \xi_j \tag{6}$$

- **If we assume $\{\xi\}_j$ (the unobserved characteristics) are just random noise**, this is a linear equation with data and demand parameters we could estimate via OLS (see lecture notes).
  - Assume total potential market. For example, total households in the US where each year each HH decides whether to buy a new car.
  - This assumption plus quantity-sold (q) enables you to construct $\{s_j\}_{j=0}^{J}$ and ultimately the left-hand side variable $\{y_i\}$.

Estimation: Idea Behind OLS

Figure: Actual versus Estimated Demand



Estimation: A Problem

- Goal: Estimate β, α via OLS

$$\log(s_j) - \log(s_0) = \beta x_j - \alpha p_j + \xi_j$$

- Valid OLS estimation requires:

  1. Homoscedasticity: $Var[\xi|x] = \sigma^2 I_J$

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     - *Violation affects standard errors. Easy fix.*

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     - *Big problem and difficult to spot without a model to frame your thoughts*

- Does "Exogeneity" hold here? Look at the Firms' profit-maximization problem.

Theory + Estimation: Firm Profit Maximization

- Assume product characteristics (x) are fixed but prices are not.
- A firm which sells product j chooses price p_j to solve:

$$\max_{p_j} \left(p_j - c_j \right) \times \overbrace{M s_j(\mathbf{p};\xi)}^{y_j(\mathbf{p};\xi)}$$

$$\Rightarrow \quad s_j(\mathbf{p};\xi) + (p_j - c_j) \times \frac{\partial s_j(\mathbf{p};\xi)}{\partial p_j} = 0$$

- So if a demand shock happens to product j (*i.e.*, change in ξ_j), firm responds by changing price.

$$\Rightarrow E[\xi|p] \neq 0$$

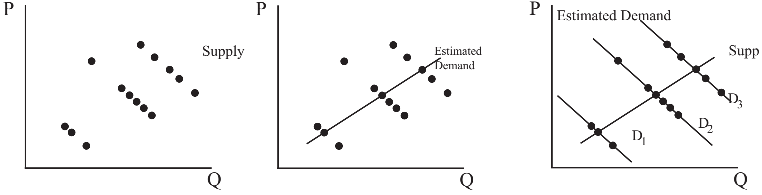
- Then the *Exogeneity* OLS assumption is violated and our OLS estimates are biased (*i.e.*, wrong).
- Further, our model enables us to sign the bias since $corr(\xi, p) > 0$:

$$\hat{\alpha}^{ols} > \alpha$$

where we α is the "true" price coefficient.

The Problem Illustrated

Figure: Demand and Demand Without Price Controls



(a) Observation (b) Estimation (c) Reality

Theory + Estimation: What to do?

- We need to isolate changes in price due to cost (shifts in supply) from changes in price due to changes in ξ (shifts in demand).
- Do this by introducing a variable correlated with changes in cost but not changes in demand. We call such a variable an **instrumental variable (IV)**.
- These can be difficult to find. Suppose we do have some IVs (call them "z"), then proceed in two steps:

  1. Regress observed price on the product characteristics (the stuff which is exogenous) and these instruments. This is like predicting price conditional on these characteristics and proxies for changes in cost (*i.e.*, the IVs).

  $$p_j = \beta x_j + \gamma z_j + \varepsilon_j$$

  2. Use the predicted prices ($\hat{p}$) in estimating demand in place of the prices from demand.

  $$\log(s_j) - \log(s_0) = \beta x_j - \alpha \hat{p}_j + \xi_j$$

- This process is known as two-stage least squares (2SLS).

Example: Demand for European Cars

- Sales of passenger cars in Spain.
- The data covers from 1995 to 2000 (1,269 observations).
- A single observation details the year, brand, model, manufacturer, price, the number of units sold, and a variety of product characteristics.

Potential Product Characteristics

- QUANTITY - Monthly registrations.
- PRICE - Market price in Euros of 1994. It includes indirect tax, transport and registration cost.
- HP - Horsepower.
- C90 - Consumption (in liters) to cover 100 km at a constant speed of 90 km/h.
- LENGTH - Length in inches.
- WIDTH - Width in inches.
- WEIGHT - Weight in pounds.
- DIESEL - Fuel engine: it is equal to one if it is a diesel car, zero if it is a gasoline car.
- NONEURO - Dummy variable: =1 for Non-European firms.
- FUELPRICE - Market price of fuel in Euros of 1994. It includes indirect taxes.
- SEGMENT - Classification at 5 market segments (e.g., sedan).

Estimating Demand

1. Use model to construct market shares by year using Q.
   - Need to take a stand on potential market $\Rightarrow s_{jt}, s_0$.
   - Assume each household considers buying a new car each year. This implies an "inside share" (*i.e.*, $\sum_{j=1}^{J} s_j$) of about 10% each year.
2. Construct $\log(s_{jt}) - \log(s_0)$.
3. Choose product characteristics to include in demand.
4. Collect and/ or construct instruments.
   - Interact average steel price over time with size and weight. Why are these valid instruments?
5. Estimate...

Hmmm... This is a Lot of Work. Why do it?

- Many interesting (*i.e.*, important) questions are difficult to analyze empirically since we lack a "natural experiment" to investigate the equilibrium implications of a policy. For example, we may want to know the implications of a policy idea before we implement it.
- By modeling consumer decisions directly and estimating consumers' (indirect) utility functions, we've created a representation of how we think an industry operates (*i.e.*, how firms and consumers make decisions).
- We can investigate how agents in the model respond to changes in their environment: *i.e.*,
  - We can add or remove a policy.
  - We can shut-down particular aspects of the model to evaluate their relative importance in determining equilibrium variables we care about (e.g., consumer welfare, firm profits).

Hmmm... This is a Lot of Work. Why do it? (cont'd)

- In each "counterfactual" we have to jointly resolve the consumers' utility-maximization problem and the firms' profit-maximization problem.
- We can also evaluation welfare implications:
  - Producer Surplus is easy since profits are easy to compute.
  - Consumer welfare is more difficult, but doable. We usually focus on compensating variation rather than the change in consumer surplus because the former accounts for income effects.

    Compensating Variation: The amount of money required to compensate a consumer for a price change where $\mathrm{CV} < 0$ means the consumer benefits from the change in prices so would be willing to pay for it. When there is no income effect, $\mathrm{CV} = \Delta \mathrm{CS}$.

Empirical Application

Application: Wollmann, T. (2018) "Trucks without Bailouts: Equilibrium Product Characteristics for Commercial Vehicles."

- Evidence: Antitrust law and regulation is mostly focused on price effects:

  Less Competition $\Rightarrow$ **Market Power** $\uparrow \Rightarrow$ **Prices** $\uparrow \Rightarrow$ **CS** $\downarrow$

- Evidence: Entry and exit of products are an important force in many markets.
- Implication: Accurately predicting changes from a merger or bankruptcy should incorporate this behavior.

Empirical Setting

- In late 2008 and early 2009, the US government invests $85 billion via the **T**roubled **A**sset **R**elief **P**rogram to bail-out domestic automakers GM, GMAC, and Chrysler.
  - GM and Chrysler commercial vehicles required $6 billion in the bailout (Author's calculation).
  - Ford drew $5.9 billion from a government-backed $9 billion line-of-credit (Forbes).
- This was a contentious decision that influenced the 2008 presidential campaign.
- Author uses data on commercial vehicle offerings between 1987 and 2012 to estimate the equilibrium impact to output and prices in an alternative world where the government had not rescued these automakers.
  - Liquidation of GM and Chrysler so these vehicles disappear.
  - Acquisition of US firms by rivals (e.g., Ford).

Empirical Approach

- Estimate demand for commercial vehicles (trucks) accounting for:
  - Horizontal differentiation (cab style, carrying load, transmission, engine-type).
  - Buyer attributes which differ by industry.

- Model is a two-stage game where each year:
  1. Firms simultaneously choose product offerings, *i.e.*, make model-level entry and exit decisions, with the understanding that their actions and their rivals' actions will impact the second stage.
  2. Firms observe product set and simultaneously choose prices.

- Econometrics: exogenous, unexpected macroeconomic shocks during the period increases the number of prospective buyers in one industry (e.g., "General Construction") but not others so demand shifts for only the subset of products preferred by that industry.

- Compare estimated equilibrium to "counterfactual" equilibria to estimate the effects of the bailout on prices, quantities, and welfare.

  Q: Was the US government's bailout of the auto industry worth the expense?

Results

- Experiment: Liquidation of GM & Chrysler.
  Mitt Romney, "*Let Detroit Go Bankrupt*"

- Let's define the following:
  - "Simple" Analysis – Solve for Bertrand Nash equilibrium prices holding the product set fixed.
  - "Complex" Analysis – Solve for Bertrand Nash equilibrium prices allowing for product entry and exit.

- Consumer impact:
  - Simple: Consumers are worse off by $253 million (compensating variation).
  - Complex: Consumers are worse off by $25 million (compensating variation).

- **Intuition:** If GM & Chrysler were to exit the market, excess demand for the types of commercial vehicles produced by these firms would lead surviving auto firms to introduce new products of similar characteristics.
  - The "Simple" analysis therefore over-states the equilibrium consumer welfare costs of allowing GM & Chrysler to fail.

Q: Was the Auto Bailout A Good Policy?

- Was the auto bailout worth it?
  - Yes, if the benefits are greater than the costs: *i.e.*,

$$\underbrace{\frac{\text{Compensating Variation}}{1-r}}_{\text{PDV Consumer Benefit of TARP}} \geq \underbrace{\text{Cost of Program (\$6 billion)}}_{\text{Financed Eventually via Taxation}}$$

  - CV is either \$253 million (simple) or \$25 million (complex).
  - The parameter $r \in (0,1)$ is the discount rate which in equilibrium is $r = \frac{1}{1+R}$ where R is the interest rate associated with a safe asset (Why?).
  - The long-run returns on U.S. bonds (*i.e.*, a safe asset) is about 4% so let's use that (*i.e.*, $r = 0.96$).
  - Bailout was good policy (LHS≈\$6.3 bn), if product entry and exit are ignored (*i.e.*, simple analysis).
  - Bailout was bad policy (LHS≈\$0.6 bn), if product entry and exit are accounted for (*i.e.*, complex analysis).
- **Big Idea:** Accounting for endogenous product entry, exit, and placement is a big deal for evaluating policy!